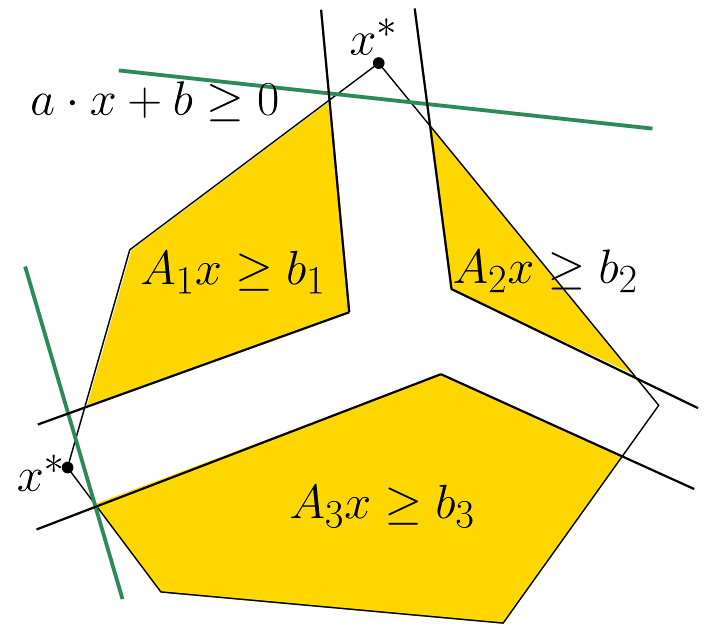
Cut Generating SDPs : A disjunctive framework for MINLPs with polynomial constraints

Amitabh Basu and Sam Burer

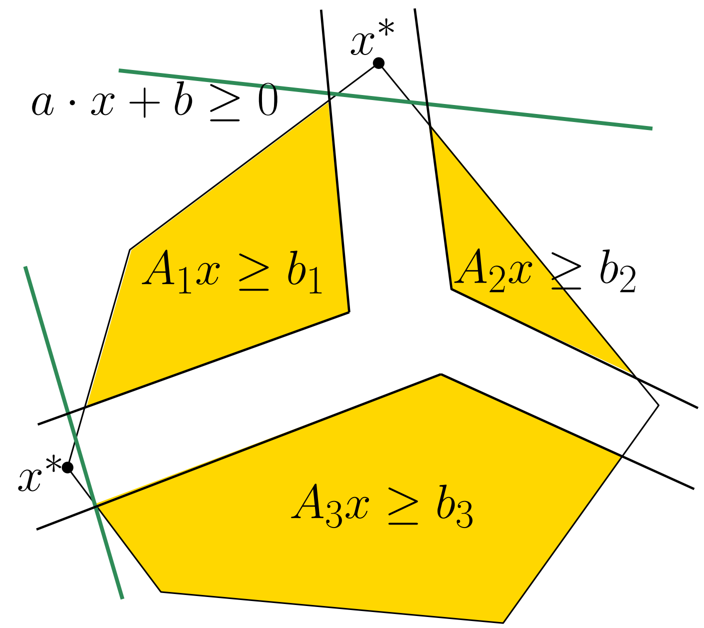
MINLP workshop, CMU Pittsburgh, June 2, 2014

Balas' CGLPs : A disjunctive framework for the polyhedral world



Balas' CGLPs : A disjunctive framework for the polyhedral world

Farkas' Lemma



CGLP

$$\min \; x^* \cdot a + b$$

$$\text{s.t. } a^T = \lambda_i^T A_i \quad i = 1, 2, 3$$

$$\lambda_i^T b_i + b \geq 0 \quad i = 1, 2, 3$$

$$\lambda_i \geq 0 \quad i = 1, 2, 3$$

$+$ Normalizing constraints

Variables : a, b, λ_i

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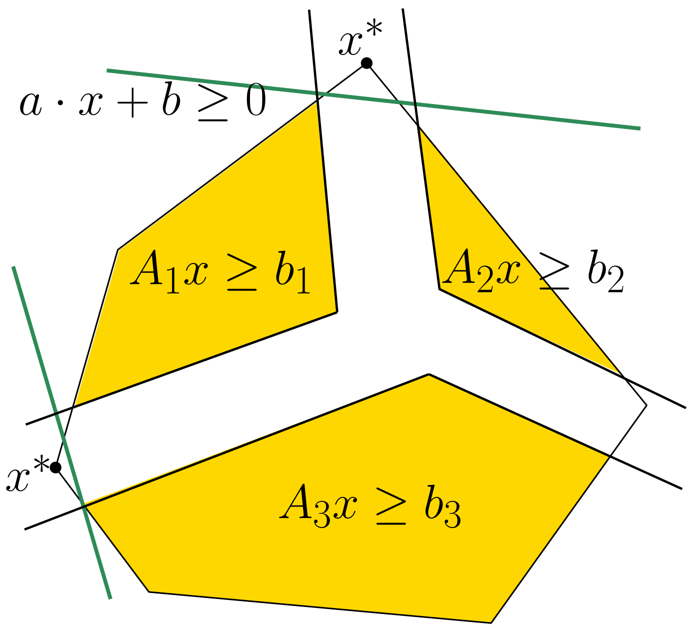

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Dual

$$A_i x_i - b_i t_i = 0 \;\; i = 1,2,3$$

$$x_1 + x_2 + x_3 = x$$

$$t_1 + t_2 + t_3 = 1$$

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Extended Formulation

Cut Generating SDPs (CGSDPs) : A disjunctive framework for the polynomial constraints

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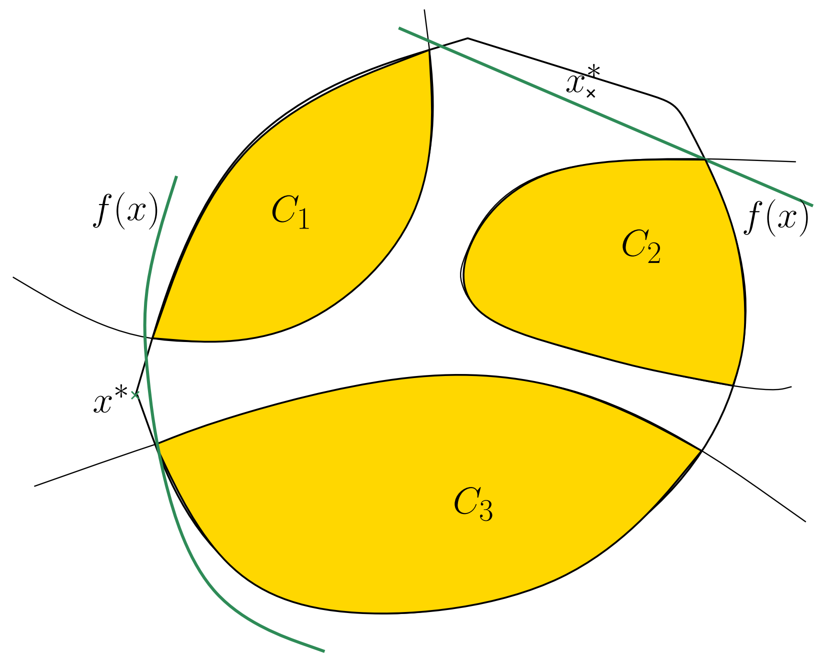

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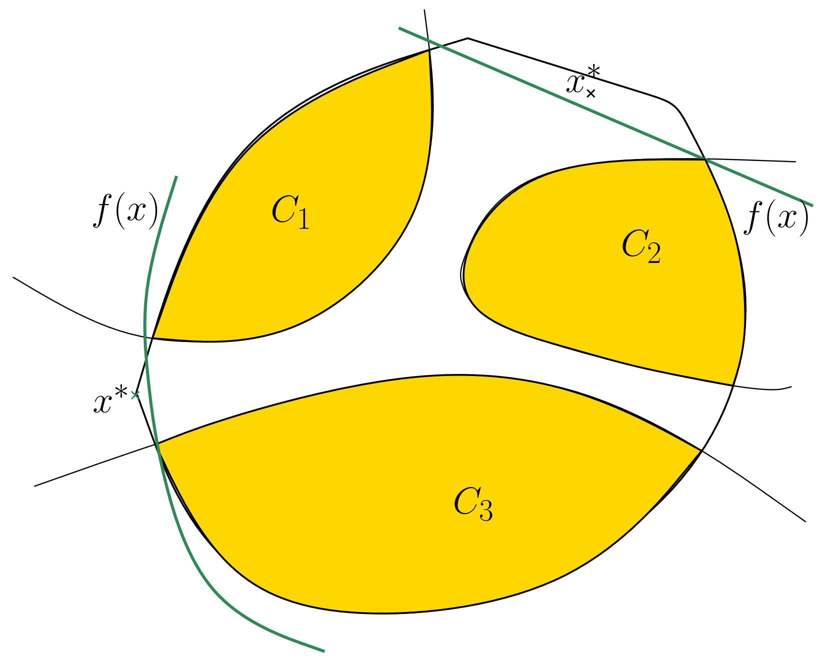
Positivstellensatz: Nonlinear Farkas' Lemma



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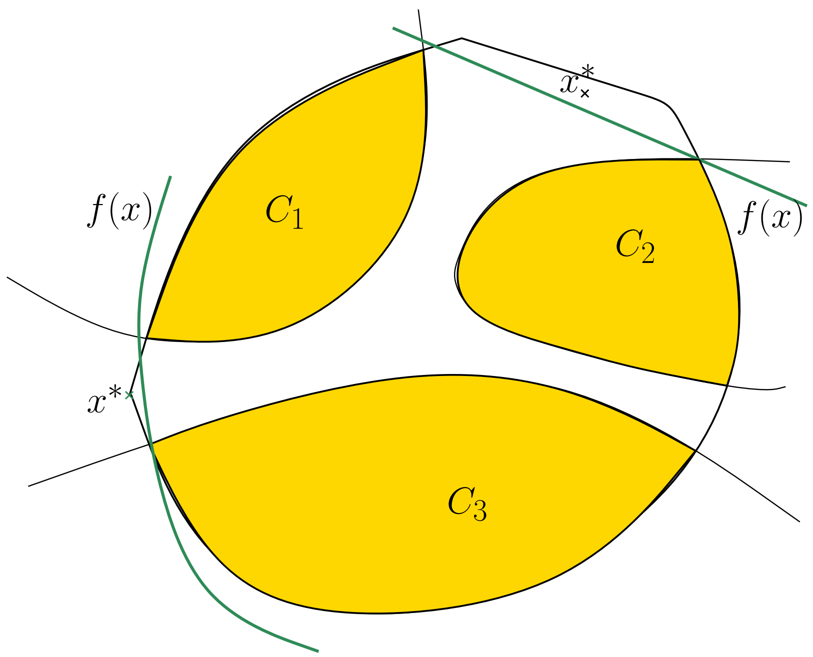

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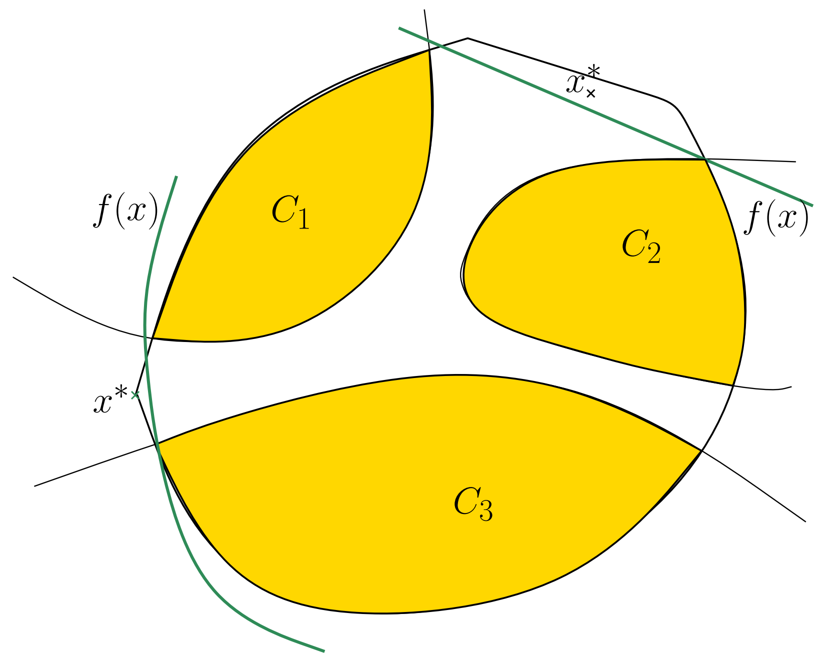

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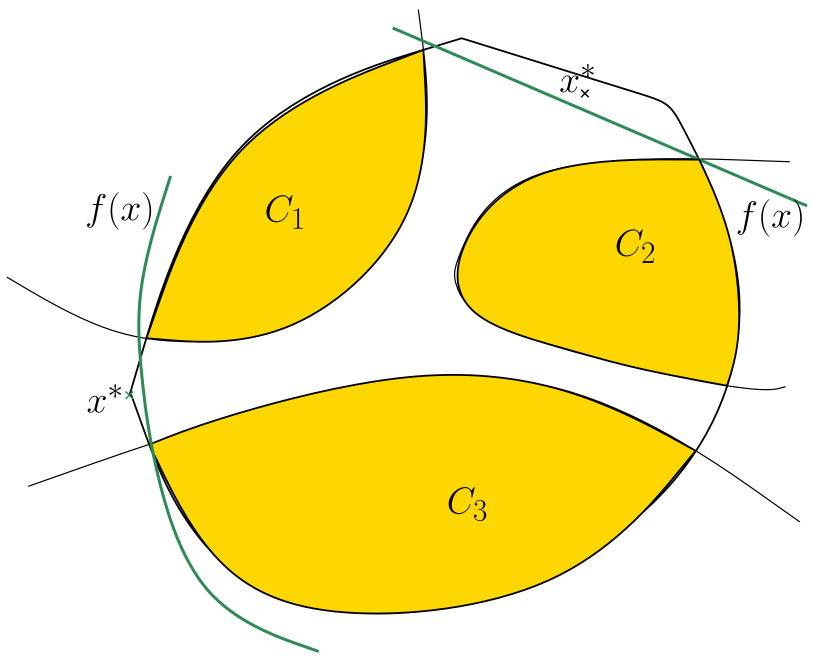

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Extended Formulation as SDP

See you at
my poster !!