Generation Investment Equilibria with Strategic Producers

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Investment Equilibria in an Oligopolistic Pool

Several
Strategic Power Producers

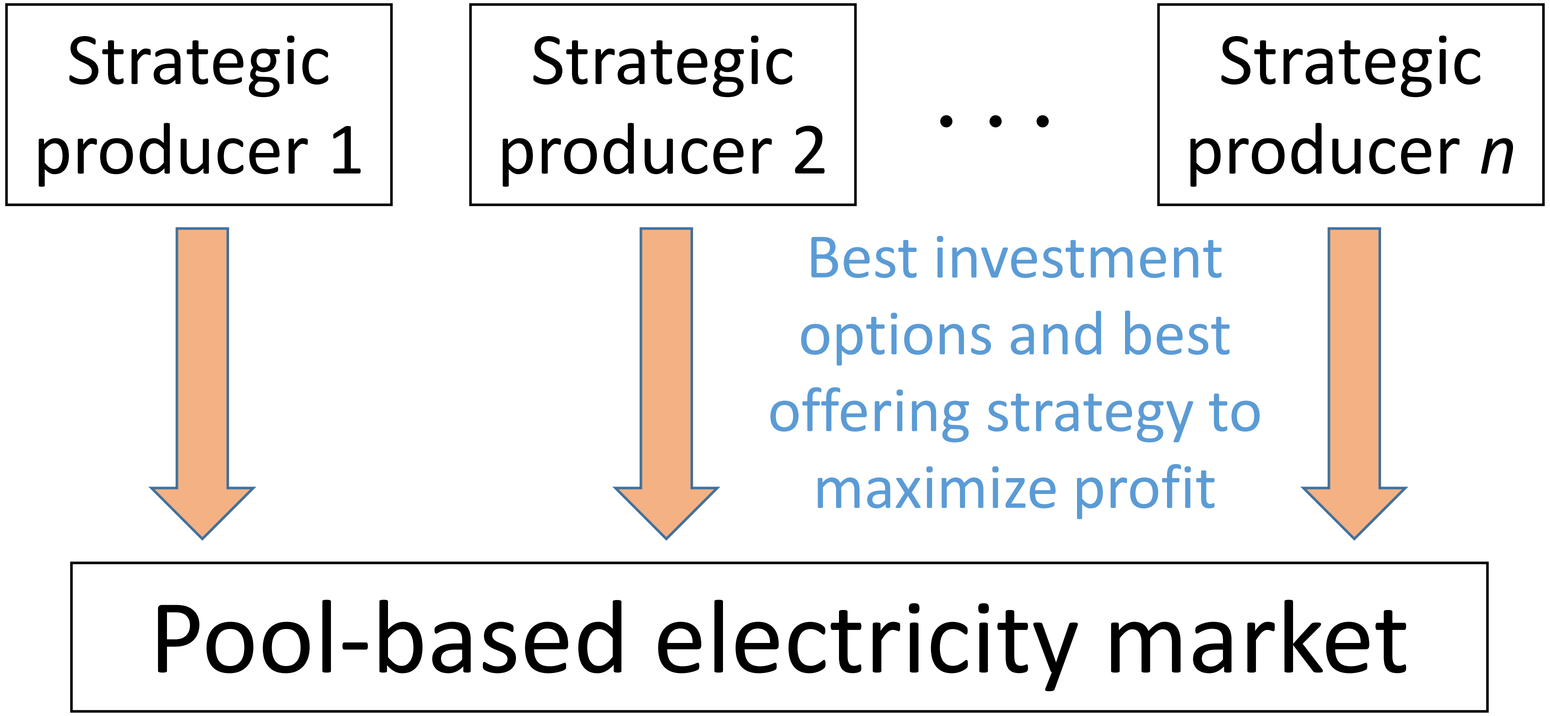
- Each producer own a significant number of production units
- These units are distributed throughout the electric energy network

Investment Equilibria in an Oligopolistic Pool

Pool-based Electricity Market

- Social welfare maximization
- DC network (1st and 2nd Kirchhoff laws) representation
- Locational marginal prices

Investment Equilibria in an Oligopolistic Pool



Is there a state in the market where no producer can increase its profit by changing unilaterally its strategy (Nash equilibrium)?

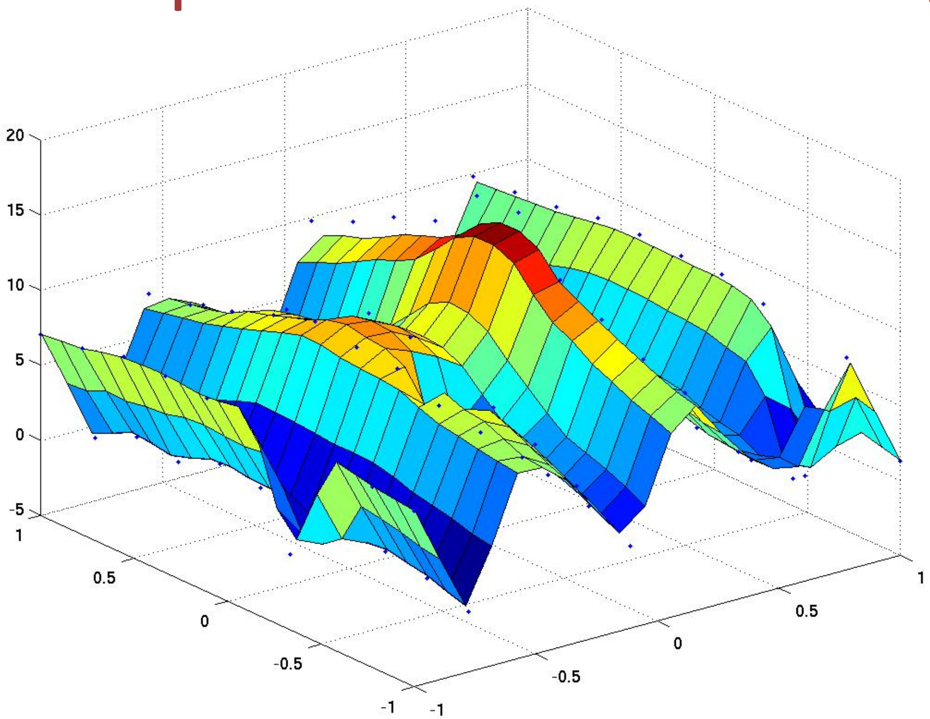
Investment Equilibria in an Oligopolistic Pool

1. The strategy of any producer is related with those of other producers by the market clearing algorithm.

2. Decisions made by one producer may influence the strategies of other producers.

3. A number of investment equilibria may exist.

Investment Equilibria in an Oligopolistic Pool



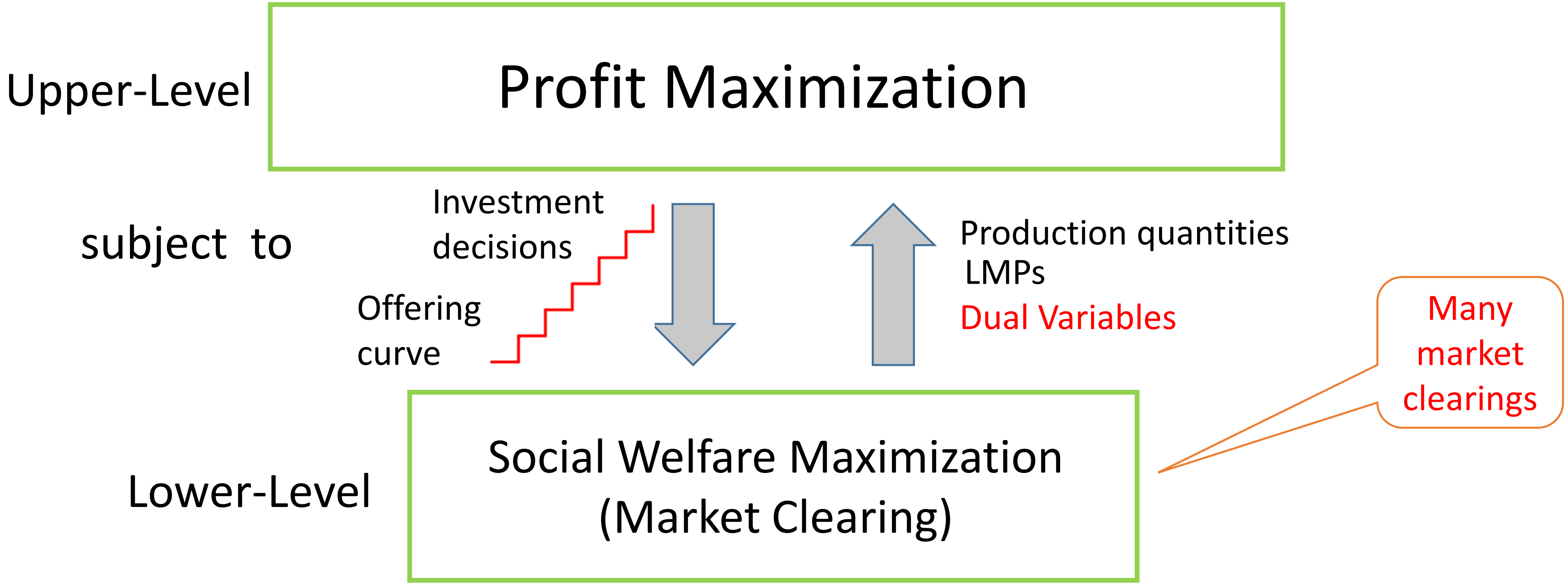
1) GNE with shared constraints: market clearing common to all producers.

2) Multiple or even infinitely many solutions.

3) Choosing a meaningful solution is not simple.

Investment Equilibria in an Oligopolistic Pool

Single-producer problem: bilevel model



MPEC

Mathematical Program with Equilibrium Constraints (MPEC)

Objective function (minimize or maximize)

subject to:

Constraints

Constraining optimization problem 1

⋮

Constraining optimization problem n

Profit maximization

Market clearing

Market clearing

MPEC

$$\text{Minimize}_{\{x\}\cup\{x^1,x^2,\lambda^1,\lambda^2,\mu^1,\mu^2\}} \quad f(\boxed{x}, x^1, x^2, \lambda^1, \lambda^2, \mu^1, \mu^2)$$

s. t.

$$h(x, x^1, x^2, \lambda^1, \lambda^2, \mu^1, \mu^2) = 0$$

$$g(x, x^1, x^2, \lambda^1, \lambda^2, \mu^1, \mu^2) \leq 0,$$

$$\left\{\begin{array}{lr} \text{Minimize}_{\boxed{x^1}} \quad f^1(x, \boxed{x^1}, x^2) & \\ \text{s. t.} & \\ \quad h^1(x, x^1, x^2) = 0 & (\lambda^1) \\ \quad g^1(x, x^1, x^2) \leq 0 & (\mu^1) \end{array}\right.$$

$$\left\{\begin{array}{lr} \text{Minimize}_{\boxed{x^2}} \quad f^2(x, x^1, \boxed{x^2}) & \\ \text{s. t.} & \\ \quad h^2(x, x^1, x^2) = 0 & (\lambda^2) \\ \quad g^2(x, x^1, x^2) \leq 0 & (\mu^2) \end{array}\right.$$

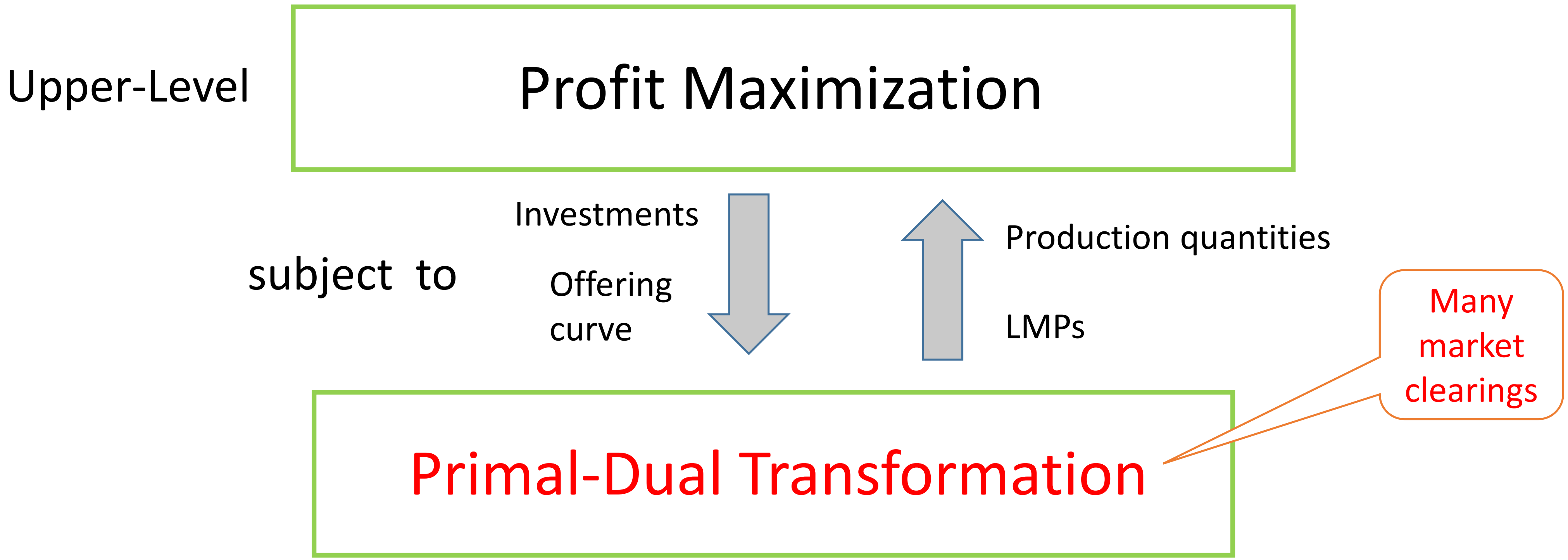
Profit maximization

Market clearing

Market clearing

Investment Equilibria in an Oligopolistic Pool

Single-producer problem: MPEC



Investment Equilibria in an Oligopolistic Pool

Single-producer problem: MPEC

Upper-Level

Profit Maximization

subject to

Investments

Offering curve

Production quantities

LMPs

Primal constraints (PC)
Dual constraints (DC)
Strong duality equality (SDE)

Many market clearings

Investment Equilibria in an Oligopolistic Pool

Multi-producer problem: EPEC

All producers

MPEC of producer 1

Minimize
Objective function of ULP 1
subject to
Constraints of UL 1

LLP 1:
PC
DC
SDE

. . .

MPEC of producer *i*

Minimize
Objective function of ULP *i*
subject to
Constraints of UL *i*

LLP *i*:
PC
DC
SDE

. . .

MPEC of producer *n*

Minimize
Objective function of ULP *n*
subject to
Constraints of UL *n*

LLP *n*:
PC
DC
SDE

Common to all producers

Investment Equilibria in an Oligopolistic Pool

Multi-producer problem: EPEC

Many LL problems (operating conditions)

MPEC of producer 1		MPEC of producer *i*		MPEC of producer *n*
Minimize Objective function of ULP 1 subject to Constraints of UL 1	• • •	Minimize Objective function of ULP *i* subject to Constraints of UL *i*	• • •	Minimize Objective function of ULP *n* subject to Constraints of UL *n*
LLP 1: PC DC SDE		LLP *i*: PC DC SDE		LLP *n*: PC DC SDE

Common to all producers

EPEC

$$\mathrm{MPEC}_1$$

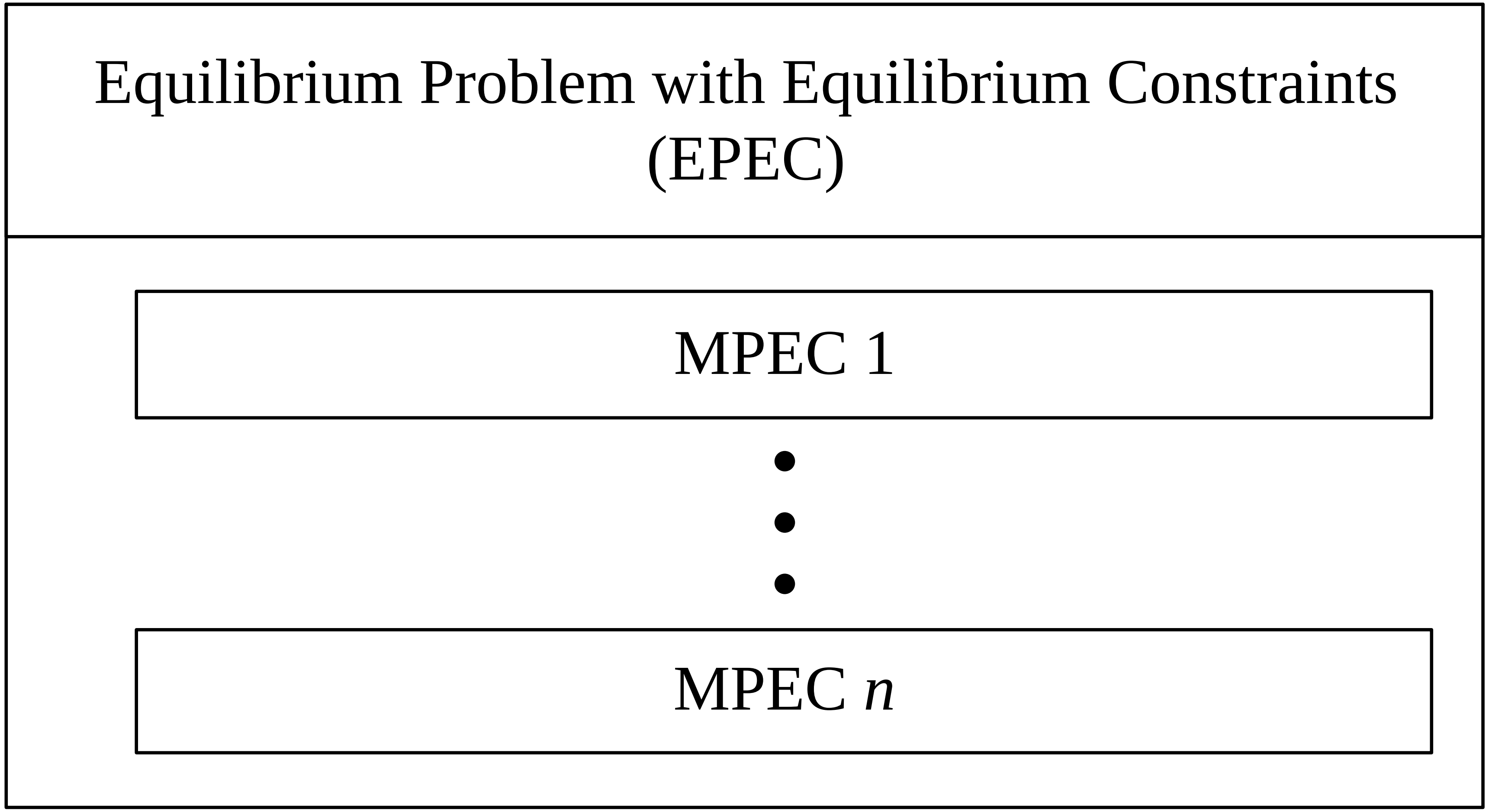
$$\vdots$$

$$\mathrm{MPEC}_i$$

$$\vdots$$

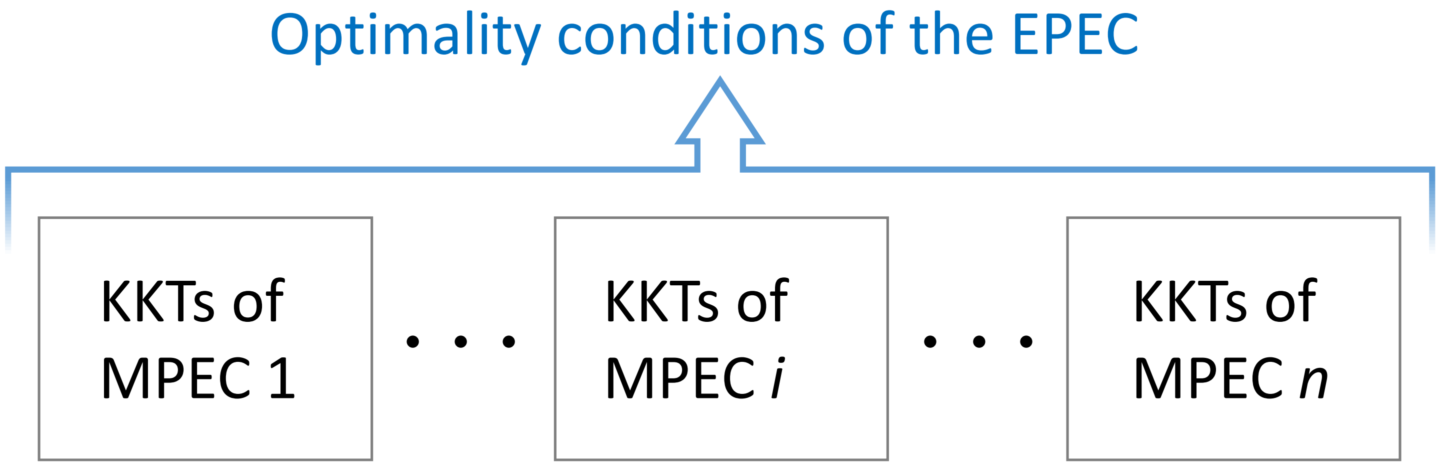
$$\mathrm{MPEC}_n$$

EPEC


Equilibrium Problem with Equilibrium Constraints
(EPEC)
MPEC 1
MPEC n

Investment Equilibria in an Oligopolistic Pool

Multi-producer problem: Optimality conditions of the EPEC



Nonlinear and highly non-convex due to both products of variables and complementarity conditions.

Investment Equilibria in an Oligopolistic Pool

KKTs of PC+DC+SDE?

- Nonlinear system of equalities and inequalities
  - MFCQ does not hold: degrees of freedom!

$\Downarrow$

Parameterization

$\Downarrow$

MIL formulation

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EPEC Linearization:

1. The complementarity conditions of the form $0 \leq a \perp b \geq 0$ (Fortuny-Amat transformation: *exact* linearization)

2. Product of variables in the strong duality equality (replaced by complementarity conditions: *exact* linearization)

3. Product of variables whose common variable is the dual variable associated with the strong duality equality (parameterization: *exact* linearization)

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Optimality conditions of the EPEC

KKTs of MPEC 1 . . . KKTs of MPEC *i* . . . KKTs of MPEC *n*

Linearization

A system of linear equalities and inequalities that involves continuous and binary variables

- Generally multiple solutions!
- How can this system be solved?

Investment Equilibria in an Oligopolistic Pool

Exploring for equilibria: Creating an optimization problem

Maximize A Meaningful Objective Function

Subject to:

System of linear equalities and inequalities that involves continuous and binary variables

Optimality conditions of the EPEC

A Meaningful Objective Function?

Investment Equilibria in an Oligopolistic Pool

Explore for equilibria: meaningful objective functions:

1. Total profit (TP) of all producers
2. Annual true social welfare (SW) considering the production costs of the generation units
3. Annual SW considering the strategic offer prices of the generation units
4. Minus payment of the demands
5. Profit of a given producer
6. Minus payment of a given demand
7. Others

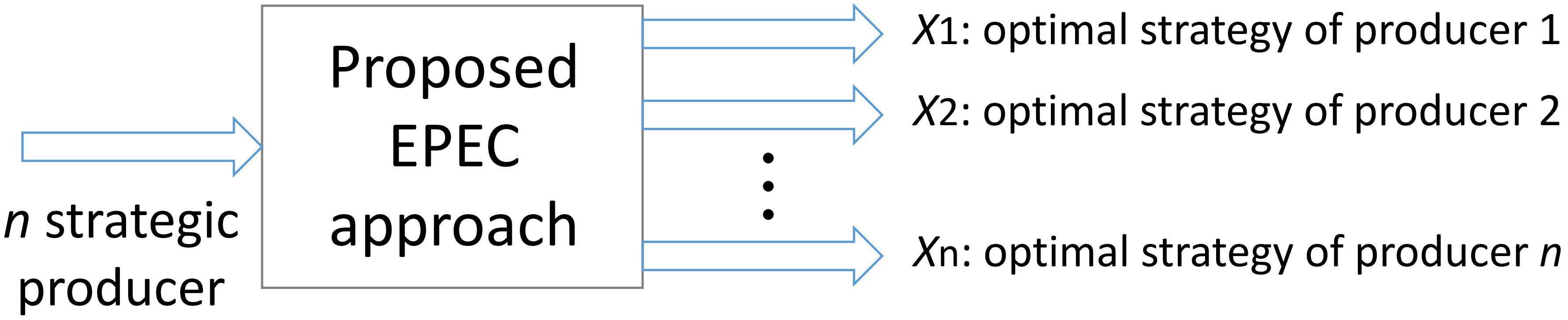
- Linear
- General market measures

Investment Equilibria in an Oligopolistic Pool

Which solutions are Nash equilibria?

A single-iteration diagonalization approach is used to verify this

Step 0:

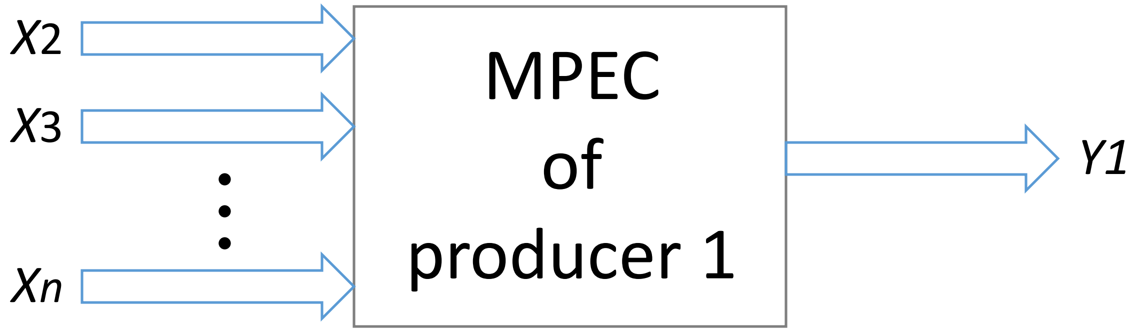


Investment Equilibria in an Oligopolistic Pool

Which solutions are Nash equilibria?

A single-iteration diagonalization approach is used

Step 1: checking optimal strategy of producer 1



Step 1 is repeated for each producer and optimal strategies *Y1, Y2, …, Yn are obtained.*

Investment Equilibria in an Oligopolistic Pool

Which solutions are Nash equilibria?

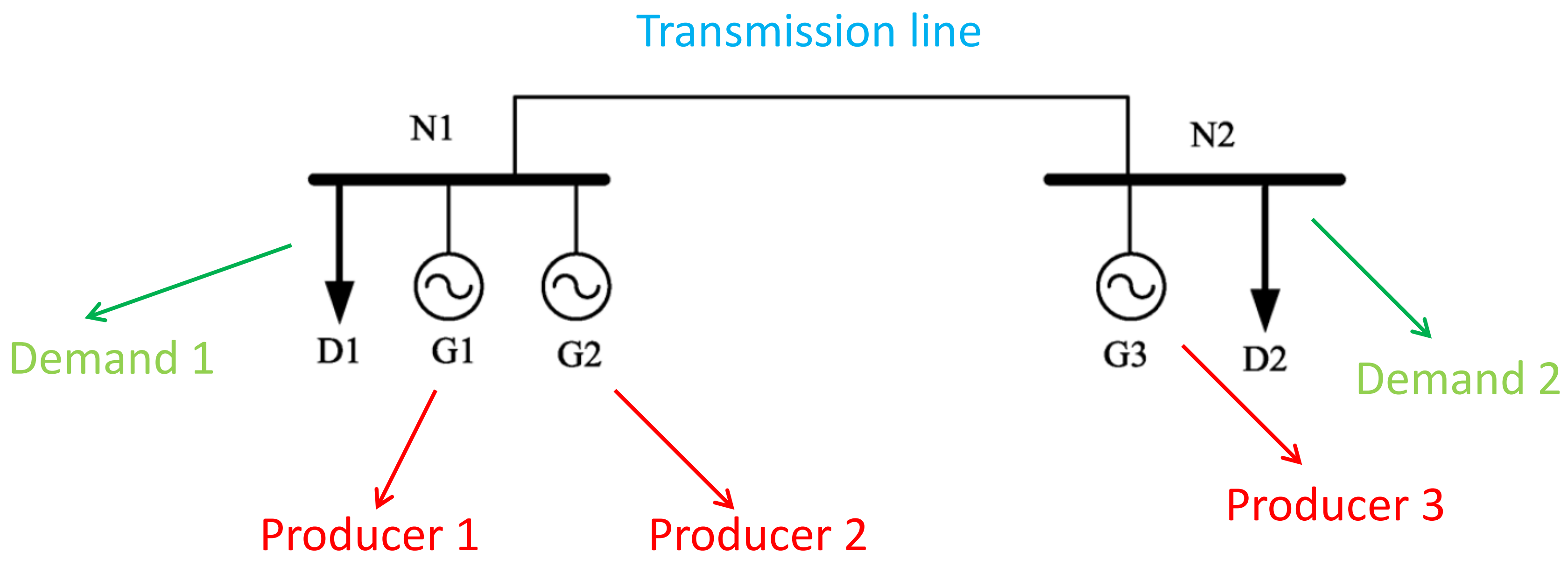
A single-iteration diagonalization approach is used

Step 2:

If $\boxed{\begin{array}{c} X1 = Y1 \\ X2 = Y2 \\ \vdots \\ Xn = Yn \end{array}}$ then, obtained solution *X1, X2, …, Xn* is a Nash equilibrium.

Investment Equilibria in an Oligopolistic Pool

Illustrative example:



Investment Equilibria in an Oligopolistic Pool

Illustrative example → cases considered:

Case A) Triopoly case: all producers are strategic
(EPEC needs to be solved)

Case B) Monopoly case: single strategic producer
(MPEC needs to be solved)

Investment Equilibria in an Oligopolistic Pool

Illustrative example → general investment results:

Case	Total capacity to be built (MW)	Total profit (M€)	Annual true social welfare (M€)
Triopoly (Max SW)	250	11.09	20.89
Triopoly (Max TP)	200	11.83	20.13
Monopoly	200	11.83	20.13

Investment Equilibria in an Oligopolistic Pool

Illustrative example → investment results for each producer (triopoly):

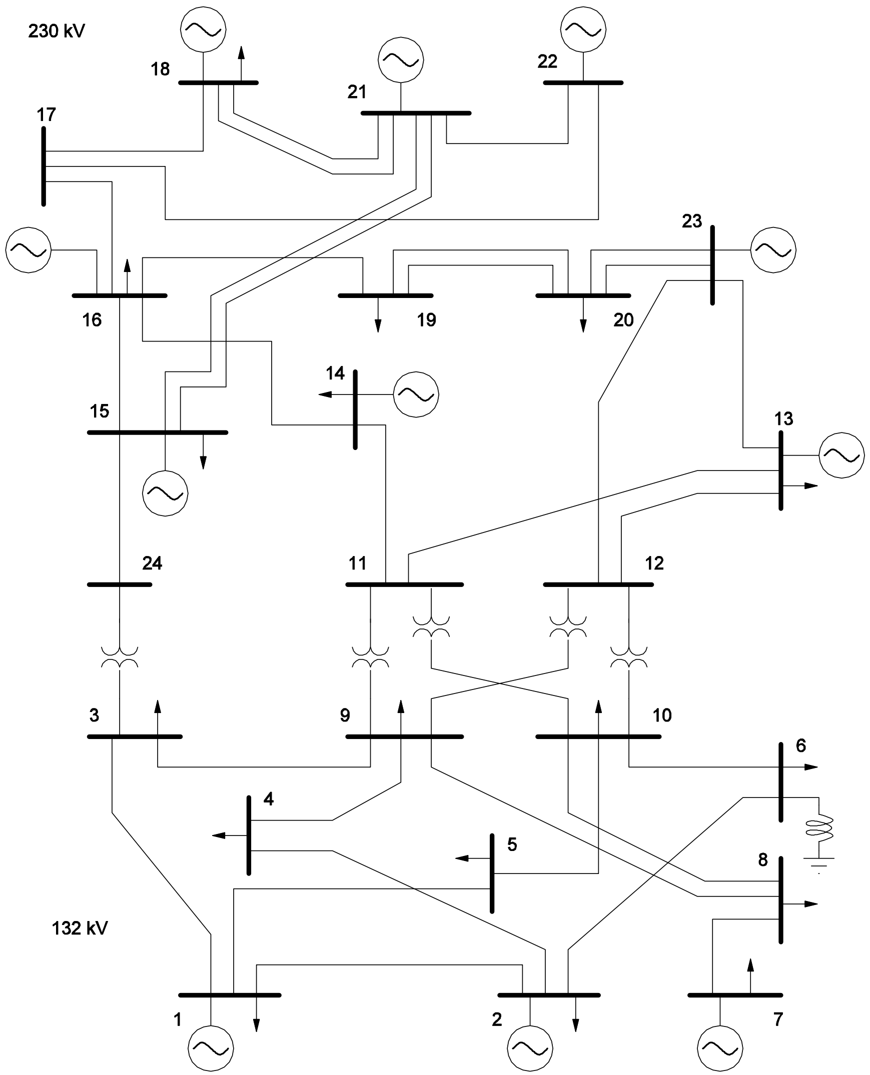
Equilibrium	E1	E2	E3
Investment of Producer 1 (MW)	200 (base)	-	-
Investment of Producer 2 (MW)	-	200 (base)	100 (base)
Investment of Producer 3 (MW)	-	-	100 (base)
Profit of Producer 1 (M€)	9.55	1.18	1.32
Profit of Producer 2 (M€)	1.18	9.55	5.30
Profit of Producer 3 (M€)	1.10	1.10	5.21
Total investment (MW)	200 (base)	200 (base)	200 (base)
Total profit (M€)	11.83	11.83	11.83

Many similar equilibria

Investment Equilibria in an Oligopolistic Pool

Case Study: one-area RTS

- **Duopoly**
- **Producer A** own all existing units of the Southern area
- **Producer B** own all existing units of the Northern area



Investment Equilibria in an Oligopolistic Pool

Case Study: one-area RTS

Case	Total capacity to be built (MW)	Total profit (M€)	Annual true social welfare (M€)
Duopoly (Max SW)	1500	113.82	175.97
Duopoly (Max TP)	1277	118.18	163.12
Monopoly	1277	118.18	163.12

Investment Equilibria in an Oligopolistic Pool

Case Study: one-area RTS

Case	Computational time (seconds)
Duopoly (Max SW)	945
Duopoly (Max TP)	164
Monopoly	1

All cases are solved using CPLEX 12.1 under GAMS on a Sun Fire X4600M2 with 8 Quad-Core processors clocking at 2.9 GHz and 256 GB of RAM.

Investment Equilibria in an Oligopolistic Pool

The following three conclusions regarding the computational burden can be drawn:

1) Each oligopolistic case needs a significantly higher computational time than any monopoly case.

2) The computational times needed for solving the duopoly cases maximizing SW are comparatively higher than the computational times required for solving the same duopoly cases but maximizing TP.

3) Congestion results in increasing the required computational time.

Conclusions

- Proposed EPEC approach → identification of meaningful equilibria
- The number of generation investment equilibria can be infinite
- Individual investment results are different across equilibria, but general investment results are generally similar
- TP is maximized→ higher profit
- SW is maximized→ higher SW
- Monopolistic market = oligopolistic market where all producers are strategic

Thank you