A decomposition framework for solving dynamic MINLP problems under uncertainty

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Scope

- Dynamic math optim under uncertainty
- A MINLP Branch-and-Fix Coordination: Exact parallel algo
- Asynchronous parallel algos:
  - Fix-and-Relax Coordination
  - Stochastic Dynamic Programming
- Some computational experience for MILP.
  Large scale instances
- Suggestion for efficiency increasing

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MINLP under uncertainty. Can be done?

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- Perhaps a strong statement: Stochastic programming machinery is ready for MILP models. See below some computational results.
- It is possible for MINLP models. Why and what:
- First, deterministic MINLP models up to the following $\#$ vars can be solved:
  - Convex MINLP: 500 (nonconvex: 100)
  - Convex NLP: 5×10^4 (nonconvex: 100)
  - Convex SOCP: 10^5 (nonconvex: 150)
  - Convex MIQP: 1000 (nonconvex: 300)
  - Convex QP: 5×10^5 (nonconvex: 300)

Source: MT 2 Mixed Integer Nonlinear Optimization by Layffer-Linderoth-Luedtke, SIAM OPT'14 (if I took it correctly).

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MINLP under uncertainty. Can be done? (c)

- Second, some types of MINLP problems exhibit this type of model:

$$\begin{aligned} & \text{máx}\, \eta \\ \text{s.t.}\quad & f_0^{\xi}(x,y) \geq \eta \\ & f_i^{\xi}(x,y) \leq 0 \quad \forall i \in \mathcal{I} \\ & x \in \{0,1\}^{n01}, \quad y^g \in \mathbb{R}^{+nc}, \end{aligned} \tag{1}$$

  where ξ represents the uncertainty in the parameters
- and third, exist a broad application field (e.g., stochastic networks, EGTCEP, SCM, PP),
  provided that non-optimal solns are also accepted:
  - Dynamic models along a horizon
  - $f(x,y)$ (smooth?) convex functions
  - non-very high number of 0-1 vars and
  - non-high number of scenarios to represent parameters' uncertainty.

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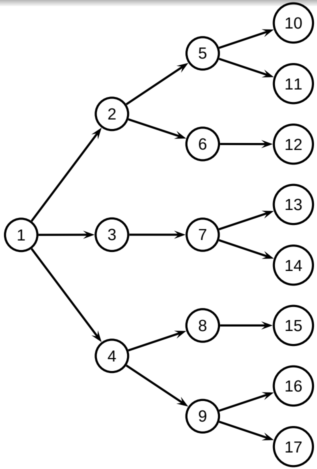
- Multistage scenario tree
- A **stage** of a given horizon is a set of consecutive time units where the realization of the uncertain parameters takes place.
- A **scenario** is a realization of the uncertain parameters along the stages of a given horizon.
- A **scenario group** for a given stage is the set of scenarios with the same realization of the uncertain parameters up to the stage.



$\Omega = \Omega^1 = \{10, 11, \ldots, 17\}$; $\Omega^2 = \{10, 11, 12\}$

$\mathcal{G} = \{1, \ldots, 17\}$; $\mathcal{G}_2 = \{2, 3, 4\}$

$\mathcal{A}^{17} = \{1, 4, 9, 17\}$

Figura: Multistage nonsymmetric scenario tree

Notation

$\mathcal{T}$, set of the T stages along the horizon.

Ω, set of scenarios.

$\mathcal{G}$, set of scenario groups, so that we have a directed graph where $\mathcal{G}$ is the set of nodes.

$\mathcal{G}^t$, set of scenario groups in stage t, for $t \in \mathcal{T}$ ($\mathcal{G}^t \subseteq \mathcal{G}$).

Ω^g, set of scenarios in group g, for $g \in \mathcal{G}$ ($\Omega_g \subseteq \Omega$).

$t(g)$, stage to which scenario group g belongs to, for $g \in \mathcal{G}$.

w^ω, likelihood or weight assigned by the user to scenario $\omega \in \Omega$.

w^g, weight assigned by modeler to scenario group $g \in \mathcal{G}$. It is computed as $w^g = \sum_{\omega \in \Omega} w^\omega$

$\mathcal{A}^g$, set of ancestor nodes (scenario groups) in the scenario tree to node (scenario) g (including itself), for $g \in \mathcal{G}$. Note: Any scenario group g from last stage is a singleton and, since $\omega \in \Omega_g$ for $g \in \mathcal{G}^T$, then let us consider $g \equiv \omega$.

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Multistage MINLP DEM: Risk neutral Compact representation

$$
\begin{aligned}
z_{RN} = \quad & \max \eta \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} w^g f_0^g(x^{g'}, y^{g'} \; \forall g' \in \mathcal{A}^g) \geq \eta \\
& f_i^g(x^{g'}, y^{g'} \; \forall g' \in \mathcal{A}^g) \leq 0 \qquad \forall i \in \mathcal{I}, g \in \mathcal{G} \\
& x^g \in \{0,1\}^{nx(g)}, \quad y^g \in \mathbb{R}^{+ny(g)} \qquad \forall g \in \mathcal{G}.
\end{aligned} \tag{2}
$$

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Scenario clustering as a framework for MINLP problem solving

Break stage t^*, t^*-decomposition

Number of clusters, $|\mathcal{C}|$,

$1 < |\mathcal{C}| \leq |\Omega|$, $|\mathcal{C}|$ MINLP submodels:

- If $t^* = 1$ then, $|\mathcal{C}| = 2$ cluster submodels.
- If $t^* = 2$ then, $|\mathcal{C}| = 4$ cluster submodels.
- If $t^* = 3$ then, $|\mathcal{C}| = 7$ cluster submodels.

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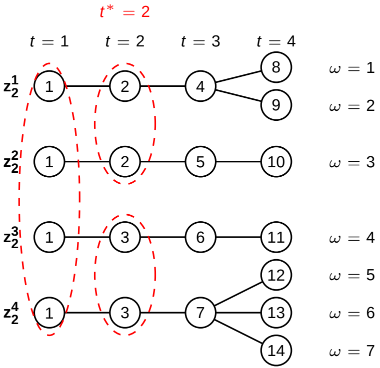

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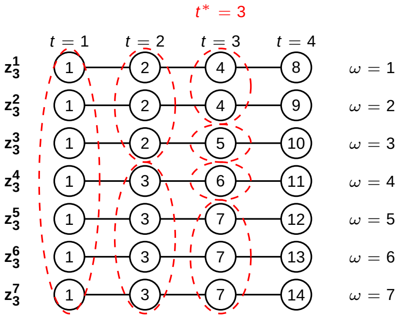

Definition

A ***break stage*** *t^* is a stage such that the set of scenario clusters is $C = |\mathcal{G}^{t^*+1}|$, where $t^* + 1 \in \mathcal{T}$. In this case, any cluster $c \in \mathcal{C}$ is induced by a group $g \in \mathcal{G}^{t^*+1}$ and contains all the scenarios belonging to that group.*

Definition

The MINLP ***scenario cluster*** *submodels are those that result from the relaxation of the NAC until break stage t^*.*

Let set $\mathcal{T} = \mathcal{T}_1 \cup \mathcal{T}_2$, where $\mathcal{T}_1 = \{1, ..., t^*\}$ and $\mathcal{T}_2 = \mathcal{T} \setminus \mathcal{T}_1$.

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Scenario clustering

Definition

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MINLP submnodel for cluster $c \in \mathcal{C}$

$$
\begin{aligned}
z_c = \quad & \max \eta_c \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}_c} w_c^g f_0^g (x_c^{g'}, y_c^{g'} \ \forall g' \in \mathcal{A}^g) \geq \eta_c \\
& f_i^g (x_c^{g'}, y_c^{g'} \ \forall g' \in \mathcal{A}^g) \leq 0 \qquad \forall i \in \mathcal{I}, g \in \mathcal{G}_c \\
& x_c^g \in \{0,1\}^{nx(g)}, \quad y_c^g \in \mathbb{R}^{+ny(g)} \qquad \forall g \in \mathcal{G}_c .
\end{aligned}
\tag{3}
$$

where $w_c^g = \sum_{\omega \in \Omega^g \cap \Omega_c} w^\omega$, such that

$w_c^g = w^{g'}$ being $g' \in \mathcal{G}^{t*+1} \cap \mathcal{G}_c$ for $g : t(g) \in \mathcal{T}_1$, and
$w_c^g = w^g$ for $g : t(g) \in \mathcal{T}_2$.

Note: **Implicit NAC (compact repr.) for each cluster.**

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NAC for linking cluster submodels

$$x_c^g - x_{c'}^g = 0,\ y_c^g - y_{c'}^g = 0 \quad \forall c, c' \in \mathcal{C}^g,\ g \in \mathcal{G}^t, t \in \mathcal{T}_1. \qquad (4)$$

Note: $\mathcal{G}_c \cap \mathcal{G}_{c'}$ is non-empty for $c, c' \in \mathcal{C}^g,\ g \in \mathcal{G}^t$ for $t \in \mathcal{T}_1$

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Cluster splitting-compact repr. of DEM MINLP (2)

$$
\begin{aligned}
z_{RN} = \quad & \text{máx}\, \eta \\
\text{s.t.} \quad & \sum_{c\in\mathcal{C}} \sum_{g\in\mathcal{G}_c} w_c^g f_0^g(x_c^{g'}, y_c^{g'}\ \forall g' \in \mathcal{A}^g) \geq \eta \\
& f_i^g(x_c^{g'}, y_c^{g'}\ \forall g' \in \mathcal{A}^g) \leq 0 \quad \forall i \in \mathcal{I}, g \in \mathcal{G}_c, c \in \mathcal{C} \\
& x_c^g - x_{c'}^g = 0, y_c^g - y_{c'}^g = 0 \quad \forall c, c' \in \mathcal{C}^g, g \in \mathcal{G}^t, t \in \mathcal{T}_1 \\
& x_c^g \in \{0,1\}^{nx(g)},\ \ y_c^g \in \mathbb{R}^{+ny(g)} \quad \forall g \in \mathcal{G}_c, c \in \mathcal{C}.
\end{aligned} \tag{5}
$$

Branch-and-Fix Coordination (BFC) methodology: Relax from the model **the explicit NAC (splitting variable repr.) between clusters**, but it algorithmically takes care of those NAC for the x- variables, see LFE-Garín-Merino-Pérez COR'10,12.

What about the explicit NAC for the y-variables?

MINLP under uncertainty. Still it can be possible?

- It depends on the modeler-driven value of break stage t^*.
- $t^* = 0$: Full compact model (2): only one scenario cluster MINLP model, too big, no NAC relaxation. difficult!
- $t^* = T - 1$: Full splitting model (5): Singleton scenario cluster MINLP models, too many NAC (4) on y-variables up to break stage t^* to relax in a first shot, risky!.
- Best value for t^*: Smallest one such the largest scenario cluster MINLP model (3) is up to the following $\#$ vars:
  - Convex MINLP: 500 (nonconvex: 100)
  - Convex NLP: 5×10^4 (nonconvex: 100)
  - Convex SOCP: 10^5 (nonconvex: 150)
  - Convex MIQP: 1000 (nonconvex: 300)
  - Convex QP: 5×10^5 (nonconvex: 300)

  Source: Layffer-Linderoth-Luedtke, SIAM OPT'14.
- Reason: The MINLP models (3) $\forall c \in \mathcal{C}$ can be solved in a **coordinated parallel mode** for (*algorithmically*) satisfying the NAC (4) on the x-variables that have been relaxed up to break stage t^*.

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BFC methodoology 1/3

- Scenario cluster based Branch-and-Fix (BF) tree is the Branch-and-Bound tree for a scenario cluster, such that the optimization of the submodel (3) for any scenario cluster $c \in \mathcal{C}$ is performed in a coordinated way with the submodels for the other clusters.
- The BFC algorithm implicitly satisfies the NAC (4) on the x-and y-variables for the set of stages in set $\mathcal{T}_2$, respectively, by using the engine of choice for solving the MINLP scenario cluster models themselves (3) at each iteration.
- On the other hand, the NAC (4) on the x-variables for the stages in $\mathcal{T}_1$ are relaxed from the original DEM (2), but their satisfaction is performed algorithmically.
- Additionally, the NAC (4) on the y-variables for the stages in $\mathcal{T}_1$ are not considered until a TNF integer set is reached.

BFC methodoology 2/3

- Scenario cluster submodels (3) $\forall c \in \mathcal{C}$ can be solved in a coordinated serial mode and even having the deterministic MINLP engine of choice as a 'subroutine' for the approach. Better proposal:
- Running in **coordinated parallel** mode the $|\mathcal{C}|$ BnC phases for solving the scenario cluster MINLP submodels (3),
- So, **imbedding the decomposition scheme into MINLP solver** for strong **interfacing** between submodels branching. Then, sharing:
  - Fixing and bound tightening on the 0-1 x vars and the continuous y vars for the stages in set $\mathcal{T}_1$.
  - Valid inequalities generation and appending where only vars for the stages in set $\mathcal{T}_1$ are involved.

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BFC methodoology 3/3

- The **branching** on the 0-1 x-variables related to the scenario groups $g \in \mathcal{G}^t$ for all stages in set $\mathcal{T}_1$ (i.e., stages **up to break stage t^***) should be **coordinated** while solving in **parallel** the $|\mathcal{C}|$ MINLP submodels, such that the replicas (one per each submodel) of each of those x-variables should be **branched in the same direction** (either 0 or 1) to algorithmically satisfying the related NAC (4)
- **Solution' feasibility**. For each coordinated feasible solution to the scenario cluster submodels (3) $\forall c \in \mathcal{C}$, a feasible solution for the original RN model (2) could be obtained by fixing the x-variables to their current 0-1 values, such that the NAC (4) on the y-variables for the stages in $\mathcal{T}_1$ are implicitly satisfied in the resulting NLP model:

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Solution’ feasibility

$$
\begin{aligned}
z_{RN}^{feas} = \quad & \max \eta \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} w^g f_0^g(x^{g'}, y^{g'} \, \forall g' \in \mathcal{A}^g) \geq \eta \\
& f_i^g(x^{g'}, y^{g'} \, \forall g' \in \mathcal{A}^g) \leq 0 && \forall i \in \mathcal{I}, g \in \mathcal{G} \\
& x^g = \hat{x}^g && \forall g \in \mathcal{G} \\
& y^g \in \mathbb{R}^{+ny(g)} && \forall g \in \mathcal{G}.
\end{aligned}
\tag{6}
$$

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Scope

- Dynamic math optim under uncertainty
- A MINLP Branch-and-Fix Coordination: Exact parallel algo
- Asynchronous parallel algos:
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  - Stochastic Dynamic Programming
- **Some computational experience for MILP. Large scale instances**
- Suggestion for efficiency increasing

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Problem #1. Randomly generated instances

- **BFC-SDC-TC strategy.** $|\mathcal{P}^{T-1}| = 2$, $|\mathcal{P}^{T}| = 4$, $|\mathcal{T}| = 5$.
- HW/SW: WS Precision T7600, Linux (version Debian2.6.32-48) with 64 bits, processor Intel(R) Xeon(R) CPU E5-2630 @ 2.3 GHz, 12 Gb of RAM and 8 threads.
- C++ experimental code.
- $|\Omega|$=574 (844) scenarios.
- P8 (P9): *m*=22650 (32184) cons, *n*01=6580 (9390) 0-1 vars, *nc*=14480 (20550) continuous vars.
- Elapsed time: 291 (1208) secs, *GG* % = 0.13 ($<$0.01) optimality gap versus plain use CPLEX.
- CPLEX v.12.5 (default options) obtains optimal sol but in 6 hours cannot prove it.
- Ref. LFE-Garín-Merino-Pérez, submitted 2014.

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Problem #2. Tactical supply chain planning under uncertainty

- **Risk averse SDC**.
- PC with a 2.5 GHz dual-core Intel Core i5 processor, 8 Gb of RAM and the operating system was OS X 10.9.
- **Metaheuristic S-SDP (Serial Stochastic Dynamic Programming)**.
- P3 (P12): $\mathcal{T}$=7 (10) periods, $\mathcal{E}$=3 (3) stages, Ω=64 (512) scenarios.
- P3 (P12): m=7827 (212544) cons, $n01$=1408 (36864) 0-1 vars, nc=4653 (124596) continuous vars.
- P3 (P12): $nprob$=544 (1258) MILP subproblems, elapsed time=610 (6540.56) secs, $GG\%$=1.82 (.) optimality gap versus plain use CPLEX v.12.5,
- CPLEX v12.5: P3 (P12): Elapsed time 17480 secs ($>$ 8 hours).
- Ref. LFE-Monge-Romero-Morales, submitted 2014.

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Problem #3. Tactical portfolio planning in the Natural Gas Supply Chain

- **Risk neutral**.
- XPRESS, CPLEX v12.2 failed to find a feas sol in several hours.
- SUN WS, 2.6Ghz, 16Gb RAM, Linux.
- **BFC-MS**.
- $|\Omega|$=1000 scenarios, m=98456 cons, n01=34680 0-1 vars, nc=22221 continuous vars.
- Elapsed time = 182 secs.
- **Optimal soln**.
- Ref. LFE-Garín-Merino-Pérez, COR'12.

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Problem #4. Multi-stage location-assignment problem under uncertainty

- **Pure combinatorial model: SLOC**.
- **Risk neutral. Metaheuristic FRC**.
- HW/SW: Core 2 Duo, 2.60Ghz, 3Gb RAM. C++ v6.0.
- CPLEX v12.3 Running out-of-memory at 38705 sec elapsed time (no soln).
- Pilot case: $|\mathcal{I}| = 15$ facilities, $|\mathcal{J}| = 75$ customers, $|\mathcal{T}| = 4$.
- $|\Omega| = 98$, $m = 203384$ cons, $n = 175525$ 0-1 vars.
- FRC elapsed time=4040 sec. Optimality GAP = 2.81 %.
- Ref. Albareda.Sambola-Alonso.Ayuso-LFE-Fernández-Pizarro, COR'13.

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Problem #5. Copper extraction planning under uncertainty in future cooper prices

CPLEX v.12.2 has problems for obtaining optimal soln.
Decomposition approaches are a must.

- **Many risk averse measures**.
- Big HW/SW platform: 2 quad-core Xeon E5450.3 3Ghz 64-bit processors with 6Mb of cache each.
- $|\Omega|$=45 scenarios, m=480490 cons, n01=167951 0-1 vars, nc=823 continuous vars.
- Elapsed time: from 398 to 29892 secs.
- **Optimal soln**.
- Ref. Alonso.Ayuso-Carvallo-LFE-Guiganrd-Pi-Puranmalka-Weintraub,
  EJOR'14.

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Problem #6. Airline Revenue Management

- **Risk neutral**.
- PC, 2.33hz, 8.5Gb RAM, Linux.
- **Continuous model**.
- Plain CPLEX v9 failed to find a feas sol in several hours.
- **Metaheuristic SDP**.
- $|\Omega|$=6561 scenarios, m=2296300 cons, nc=2624400 continuous vars.
- Elapsed time = 71 secs.
- Optimality GAP=1.22 %.
- Ref. LFE-Monge-Romero.Morales-Wang, TS'13.

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Problem #7. Production planning under uncertainty

- Serial, Inner, Outer, Outer-Inner **asynchronous parallel** metaheuristic SDP (Stochastic Dynamic Programming).
- **Risk neutral**. CPLEX v12.5 for solving independent scenario cluster MILP subproblems.
- MPI: Message Passing Interface.
- Big computing cluster, SGI/IZO-SGIker at UPV/EHU, used 16 xeon cores (8 or 12 treads each), 48 Gb each.
- $\mathcal{T}$=16 periods, Randomly generated Ω=7766.
- P86 (P85): *m*=5.56 (57.8) **M** cons, *n*01=1.41 (15.04) **M** 0-1 varis, *nc*=3.49 (38.5) **M** continuous vars.
- P86 (P85): *nprob*=28997 (5177) MILP subproblems, elapsed time=978 (26180) secs.
- P-SDP: incumbent sol value with *GG* %=0.16 (.). Effciency=61.64 (89.15) %.
- Plain CPLEX: running out of memory (35Gb) after 8274 secs, sol value with *OG* %=0.78 (.) optimality gap.
- Ref. Aldasoro-LFE-Merino-Pérez, submitted 2014.

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Suggestion for efficiency increasing

- Imbedding decomposition scheme into MINLP solver.
- Running in coordinated parallel mode the BnC phases of the scenario cluster MINLP submodels (3).
- Strong interfacing between submodels branching. So, sharing:
  - Fixings and bound tightening on the 0-1 x vars and the continuous y vars for the stages in set $\mathcal{T}_1$.
  - Valid inequalities generation and appending where only vars for the stages in set $\mathcal{T}_1$ are evolved.

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