Analyzing the computational impact of individual MINLP solver components

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joint work with Stefan Vigerske

Zuse Institute Berlin · Matheon · Berlin Mathematical School

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ZIB – Fast Algorithms, Fast Computers

Zuse Institute Berlin is a research institute and computing center of the State of Berlin with research units:

- Numerical Analysis and Modeling
- Visualization and Data Analysis
- Optimization: Energy–Traffic–Telecommunication–**Linear and Nonlinear IP**
- Scientific Information Systems
- Computer Science and High Performance Computing

What is SCIP?

SCIP (Solving Constraint Integer Programs) ...

- integrates
  - CP features (domain propagation)
  - MIP features (cutting planes, LP relaxation)
  - SAT-solving features (conflict analysis, restarts)
- is a branch-cut-and-price framework
- has an modular structure
- can be extended via plugins
- is free for academic purposes
- and is available in source-code under `http://scip.zib.de`
- provides a full-scale MIP and **MINLP solver**

Outline

Analyzing MINLP solver components

Benchmarking methodology

Separation

Reformulation

Primal Heuristics

Tree search

Propagation

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789 choose 475

Collection of 789 publicly available MINLP instances

- ▷ MINLPLib2 α: MINLPLib+minlp.org+Bonmin+...

Hardware

- ▷ Dell PowerEdge M1000e, 48 GB RAM, Intel Xeon X5672@3.2 GHz

Software

- ▷ SCIP 3.1.0.1
- ▷ SoPlex 2.0
- ▷ Ipopt 3.11.8
- ▷ CppAD 20140000.1

475 test instances, 15 settings, 1 hour time limit

- ▷ 314 instances not solved by default within 2 hours

Averaging over heterogeneous test sets

Instances vary widely in size, nonlinearity, . . .

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Instances vary widely in size, nonlinearity, . . . , **time to optimality**

- arithmetic average: dominated by large times
- geometric average: weights trivial and hard instances equally
- shifted geometric average: which shift?

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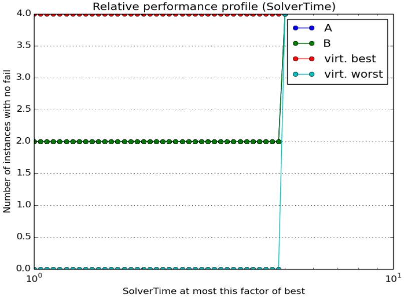
Some results are not distinguished by performance profiles alone:

inst	A	B
1	10s	2s
2	10s	2s
3	10s	50s
4	10s	50s



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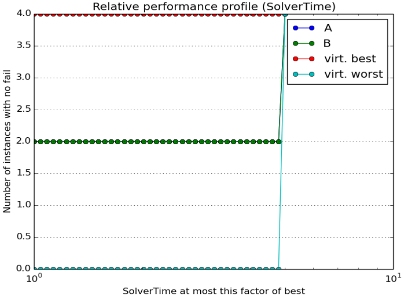
inst	A	B
1	5x	1x
2	5x	1x
3	1x	5x
4	1x	5x



Averaging over heterogeneous test sets

Instances vary widely in size, nonlinearity, ..., time to optimality

- arithmetic average: dominated by large times
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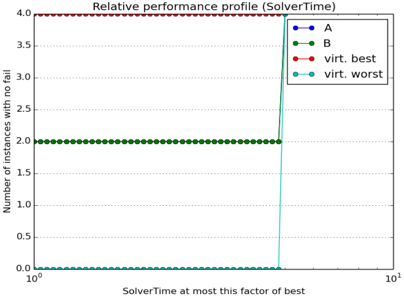
- shifted geometric average: which shift?

Some results are not distinguished by performance profiles alone:

inst	A	B
1	10s	2s
2	20s	100s
3	50s	10s
4	100s	500s



Filtered Performance Diagrams

Gradually exclude instances **solved by A and B** and compute speedup:

$$t \mapsto \frac{\mu(\{t_{A,i} : \max\{t_{A,i}, t_{B,i}\} \geq t\})}{\mu(\{t_{B,i} : \max\{t_{A,i}, t_{B,i}\} \geq t\})}$$

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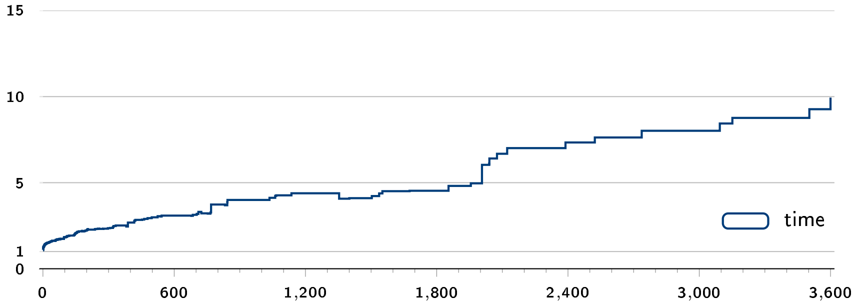
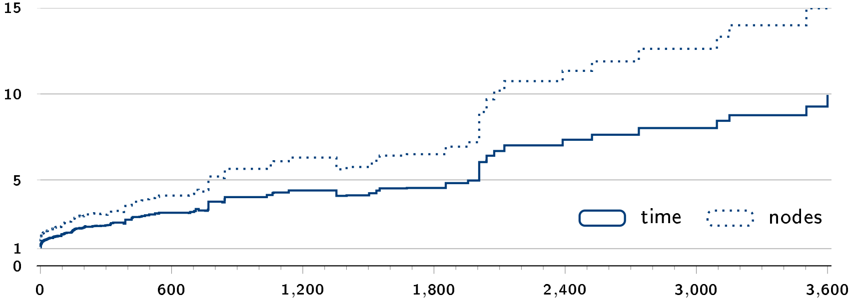


[See also Achterberg and Wunderling 2013]

Filtered Performance Diagrams

Gradually exclude instances **solved by A and B** and compute speedup:

$$t \mapsto \frac{\mu(\{N_{A,i} : \max\{t_{A,i}, t_{B,i}\} \geq t\})}{\mu(\{N_{B,i} : \max\{t_{A,i}, t_{B,i}\} \geq t\})}$$



[See also Achterberg and Wunderling 2013]

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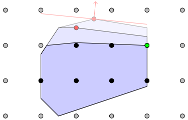
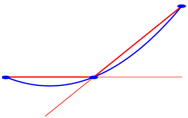
MIP cutting planes

- general: Gomory, cMIR, $\{0, \frac{1}{2}\}$-cuts, ...
- problem-specific: knapsack, clique, multi commodity flow, ...



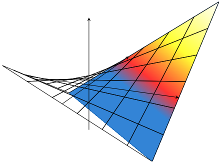
Gradient cuts for convex terms

- feasibility enforced without branching
- exploit integer information for univariate convex terms



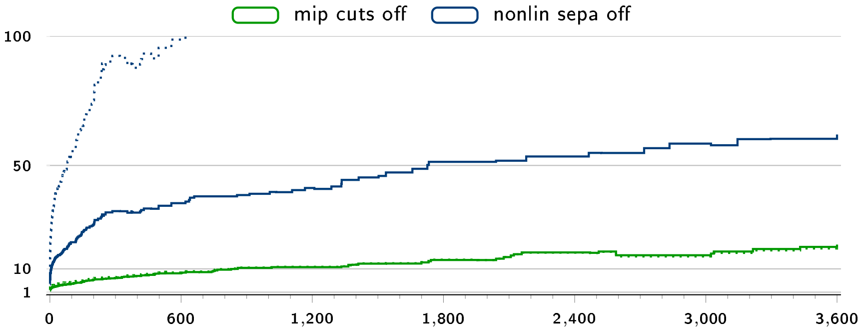
Convex underestimators for nonconvex terms

- secant, signed power, McCormick, ...



Alternative setting: off during fractional branching

Separation



		all		maxtime ≥ 100	
setting	solved	time	nodes	time	nodes
mip cuts off	−39	+65%	+107%	+333%	+395%
nonlin sepa off	−102	+302%	+695%	+1964%	+5569%

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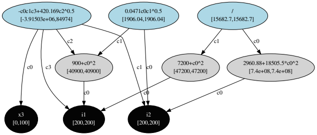
Expression graph reformulation

- ▷ decompose into Smith normal form
- ▷ identify common terms
- ▷ merge expressions, e.g., polynomials



Products with binary variables

- ▷ linearize using big-M

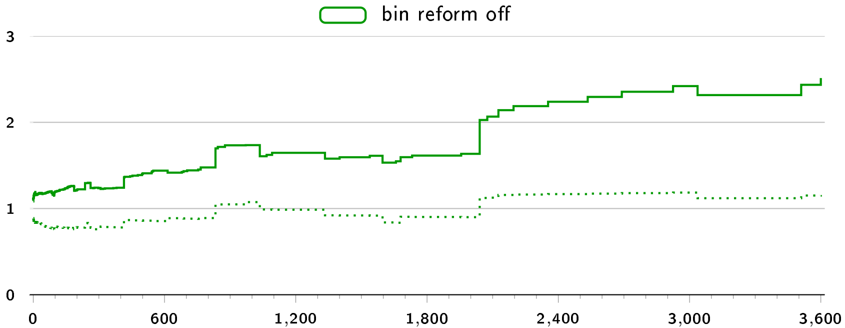
$$x \cdot \sum_k a_k y_k \quad \text{with} \quad x \in \{0,1\}$$

$$\downarrow$$

$$M^L x \le w \le M^U x,$$

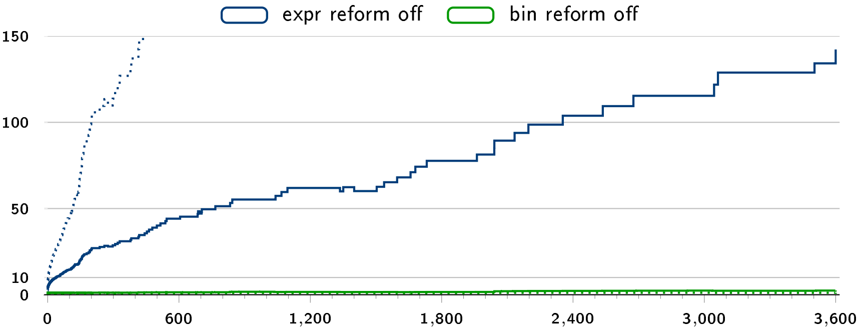
$$\sum_k a_k y_k - M^U(1-x) \le w \le \sum_k a_k y_k - M^L(1-x)$$

Reformulation



		all		maxtime $\geq$ 100	
setting	solved	time	nodes	time	nodes
expr reform off	−69	+160%	+322%	+1386%	+3631%
bin reform off	−9	+8%	−11%	+20%	−21%

Reformulation



		all		maxtime ≥ 100	
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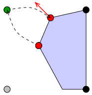
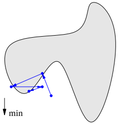
Besides waiting for feasible LP solutions . . .



Standard MIP heuristics applied to MIP relaxation

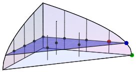
- ▷ rounding, diving, feasibility pump, . . .

NLP local search



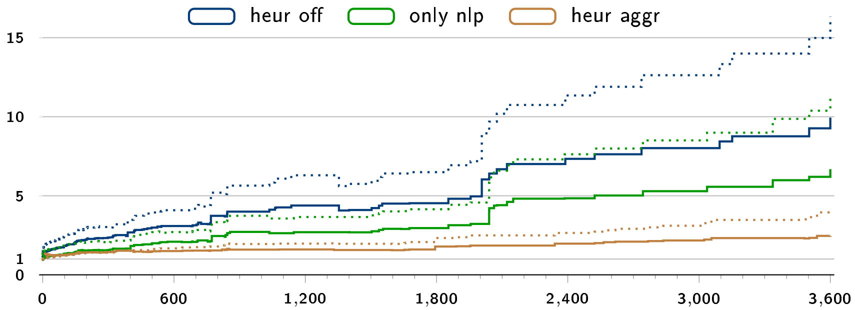
- ▷ for integer and LP feasible solutions
- ▷ fix integers and solve remaining NLP

MINLP heuristics



- ▷ NLP diving
- ▷ RENS [Berthold 2013]
- ▷ Undercover [Berthold and G. 2013]
- ▷ . . .

Primal Heuristics



		all		maxtime ≥ 100	
setting	solved	time	nodes	time	nodes
heur off	−19	+7%	+36%	+84%	+144%
only nlp	−11	−4%	+22%	+33%	+22%
heur aggr	−2	+27%	−4%	+28%	+86%

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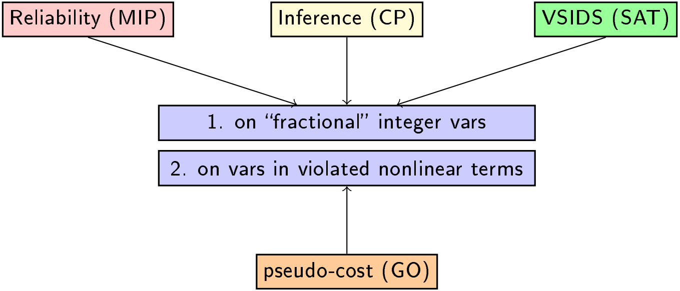
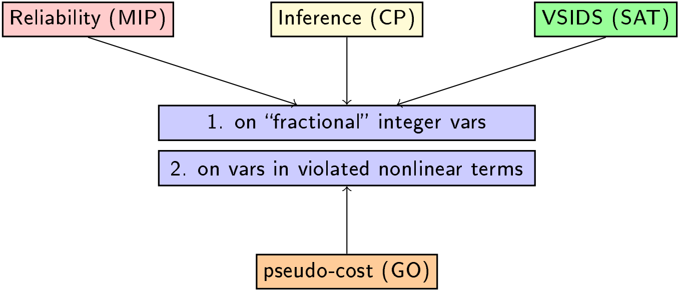
Branching



Reliability (MIP)
Inference (CP)
VSIDS (SAT)
1. on "fractional" integer vars
2. on vars in violated nonlinear terms
pseudo-cost (GO)

Branching

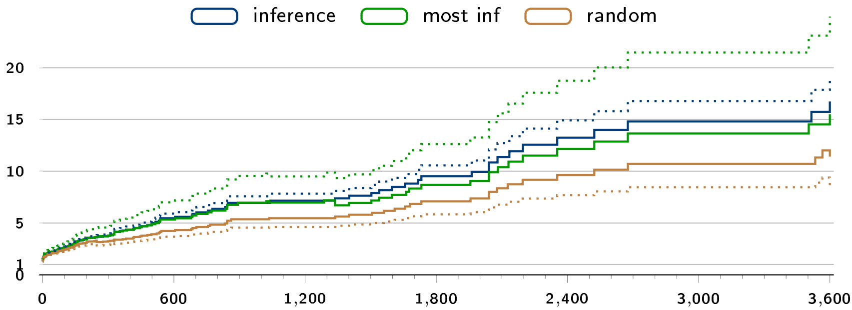


Alternative settings for spatial branching

- inference, most infeasible, random

[See Tawarmalani and Sahinidis 2002, Achterberg and Berthold 2009, Belotti et al. 2009, . . .]

Spatial Branching



		all		maxtime $\geq$ 100	
setting	solved	time	nodes	time	nodes
inference	−27	+31%	+34%	+167%	+176%
most inf	−24	+30%	+38%	+165%	+209%
random	−24	+30%	+28%	+145%	+130%

Node selection

Tasks

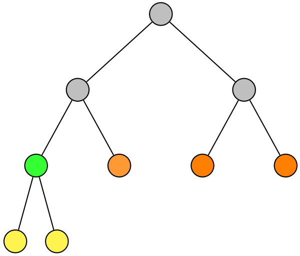
- improve primal bound
- keep computational effort small
- improve global dual bound

Best estimate with plunging

- select node Q with best/minimal (pseudo cost) estimate value for feasible solution quality

$$\bar{z}_Q + \sum_{k : \bar{x}_k \, fractional} \min\{\Psi^- f^-, \Psi^+ f^+\}$$

- plunge

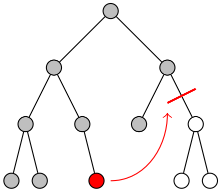


Alternative setting: breadth first search

Conflict analysis/nogood learning

Analyse reason for pruning a node

- branchings and propagations
- infeasible and bound exceeding LP relaxation: dual ray heuristic
- derive short nogoods/conflict constraints

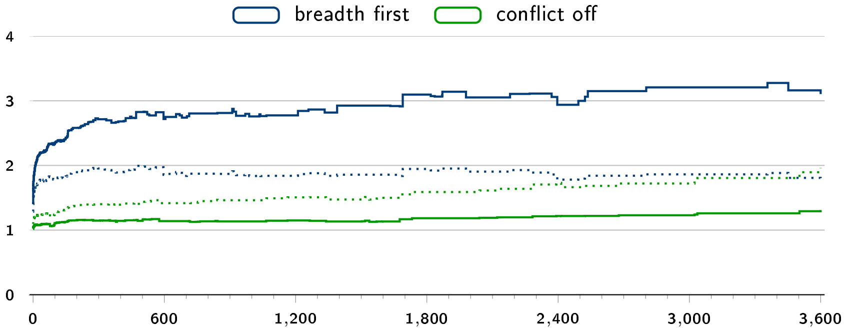


$$x_1 - x_3 \leq 0$$

Use subsequently

- to cut off other nodes
- to enable further propagations
- for VSIDS in branching

Node selection & conflict analysis



		all		maxtime ≥ 100	
setting	solved	time	nodes	time	nodes
breadth first	−22	+42%	+29%	+136%	+81%
conflict off	−2	+2%	+9%	+11%	+27%

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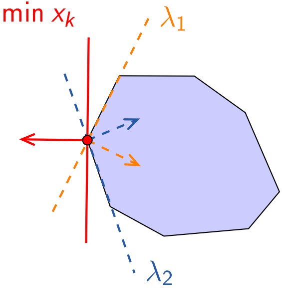
Bound tightening/propagation

Particularly important for nonconvex MINLP

- branching on continuous variables/infinite domains
- tight domains $\rightsquigarrow$ tight relaxation

Primal and dual reductions

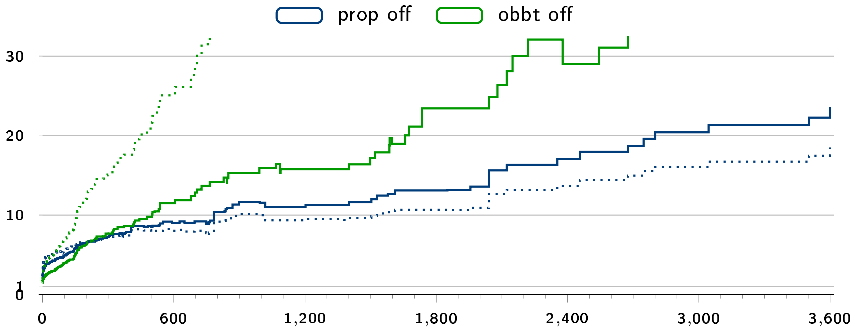
- reduced cost
- probing on binaries
- FBBT: feasibility-based bound tightening
- OBBT: optimization-based bound tightening and Lagrangian variable bounds:



$$x_k \geq \sum_{i:r_i>0} r_i \ell_i + \sum_{i:r_i<0} r_i u_i + \mu z^* + \lambda^T b$$

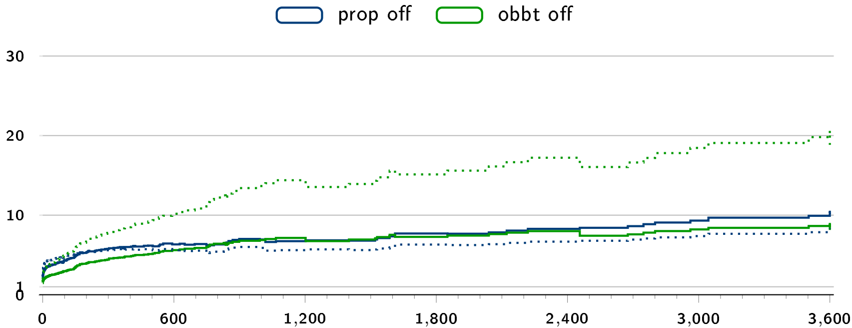
[Ryoo and Sahinidis 1996, Belotti et al. 2009, G. and Weltge 2013, ...]

Bound tightening/propagation



		all		maxtime ≥ 100	
setting	solved	time	nodes	time	nodes
prop off	−48	+90%	+129%	+397%	+461%
obbt off	−25	+47%	+93%	+303%	+607%

Bound tightening/propagation



		all		maxtime ≥ 100	
setting	solved	time	nodes	time	nodes
prop off	−48	+90%	+129%	+332%	+378%
obbt off	−25	+47%	+93%	+198%	+396%

Summary

		all		maxtime $\geq$ 100	
setting	solved	time	nodes	time	nodes
nonlin sepa off	−102	+302%	+695%	+1964%	+5569%
expr reform off	−69	+160%	+322%	+1386%	+3631%
prop off	−48	+90%	+129%	+397%	+461%
mip cuts off	−39	+65%	+107%	+333%	+395%
inference branching	−27	+31%	+34%	+167%	+176%
obbt off	−25	+47%	+93%	+303%	+607%
most inf branching	−24	+30%	+38%	+165%	+209%
random branching	−24	+30%	+28%	+145%	+130%
breadth first	−22	+42%	+29%	+136%	+81%
heur off	−19	+7%	+36%	+84%	+144%
heur only nlp	−11	−4%	+22%	+33%	+22%
bin reform off	−9	+8%	−11%	+20%	−21%
heur aggr	−2	+27%	−4%	+28%	+86%
conflict off	−2	+2%	+9%	+11%	+27%

Take-away messages

- SCIP is a global solver for nonconvex MINLPs
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- crucial components
  1. nonlinear separation
  2. expression graph reformulation
- add-on components
  1. propagation
  2. MIP cutting planes
  3. branching

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Thank you very much for your attention!