POOLING PROBLEM: NEW IDEAS ON LOWER & UPPER BOUNDS

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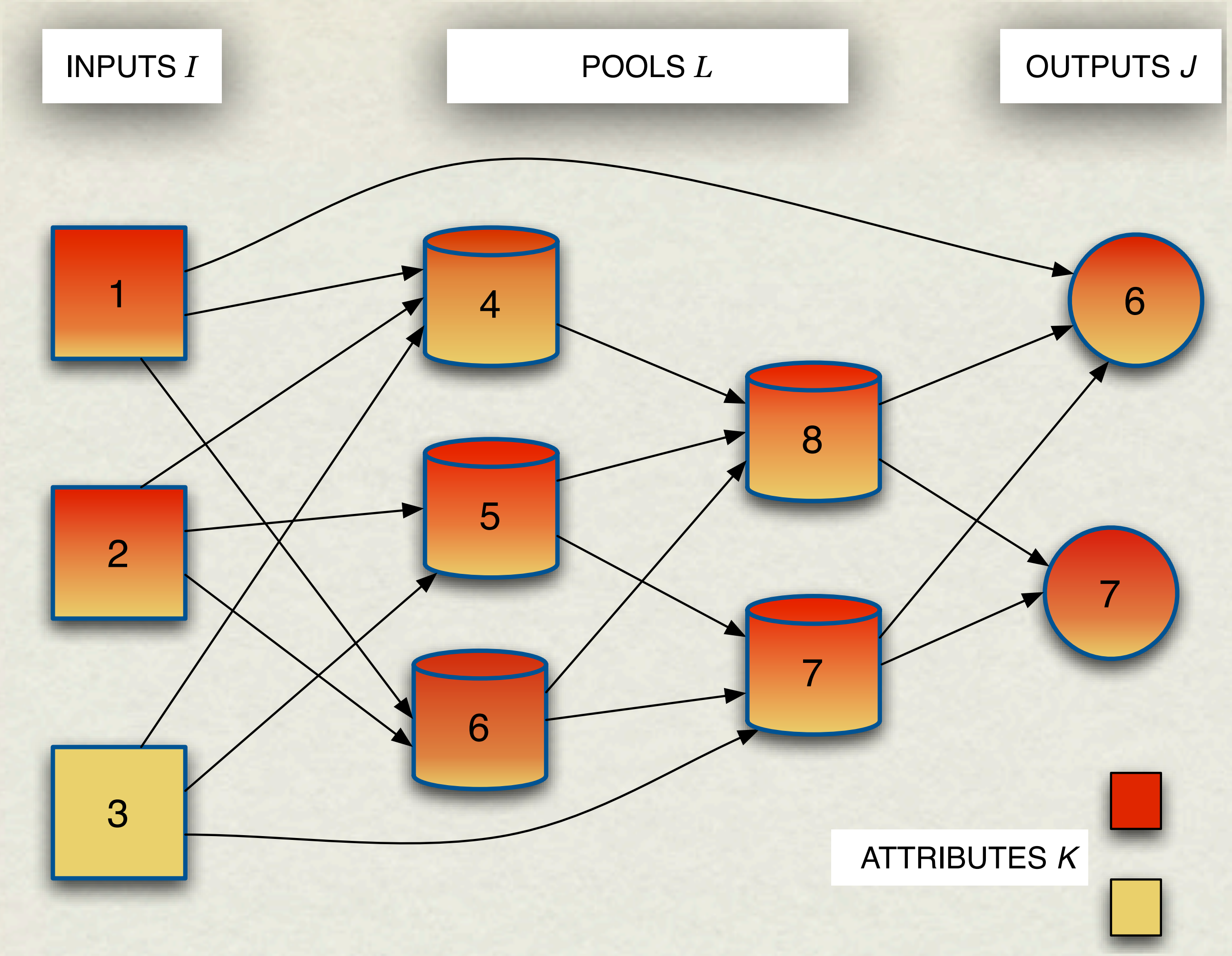
THE POOLING PROBLEM

- Type of multicommodity flow problem on tripartite graph



Attributes must satisfy lower/upper bounds at each output

- Bilinear equality and inequality constraints
- MILP relaxations due to piecewise linear estimators of bilinear terms
- MILP restrictions obtained by fixing subset of variables

CONTRIBUTIONS

- *New* family of network flow *MILP restrictions*
- *Theoretical analysis* for standard problems: Let n = # of output nodes, z^* = global optimal value.

> *Theorem 1*. For any pwl MILP relaxation S, let z^S be the optimal value of this MILP. Then,
>
> 1. $z^* \leq z^S \leq n z^*$
> 2. For any $\varepsilon > 0$, there exists a problem instance with $z^S \geq (n-\varepsilon) z^*$

> *Theorem 2*. For any $\tau \in \mathbb{Z}_{++}$, $\gamma \in \mathbb{R}^\tau$ s.t. $\sum_t \gamma_t = 1$, $\gamma \geq 0$, there is a MILP restriction **PQ**(τ,γ) with value $z(\tau,\gamma)$ s.t. $n z(\tau,\gamma) \geq z^*$ and this bound is tight for rational γ

- Empirically, the new MILPs yield *extremely good feasible solutions* on large-scale test instances