Solving MINLPs with BARON

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MIXED-INTEGER NONLINEAR PROGRAMS

$$\begin{aligned} \min \quad & f(x, y) \\ \text{s.t.} \quad & g(x, y) \leq 0 \\ & x \in R^n, \ \ y \in Z^p \end{aligned}$$

- *f, g* **are factorable functions that may be nonconvex.**

BOUNDING FACTORABLE PROGRAMS

(Ryoo and Sahinidis, 1996; Similar to McCormick, 1976)

Introduce variables for intermediate quantities whose envelopes are not known

$$f(x,y,z,w) = \sqrt{\exp(xy + w\log w)z^3}$$

$$t_1 = xy$$

$$t_2 = \log w$$

$$t_3 = wt_2$$

$$t_4 = t_1 + t_3$$

$$t_5 = \exp(t_4)$$

$$t_6 = z^3$$

$$t_7 = t_5 t_6$$

$$t_8 = \sqrt{t_7}$$

EXPLOITING CONVEXITY

(Khajavirad and Sahinidis, 2014)

Recognize convex subexpressions

$$f(x,y,z,w) = \sqrt{\exp(xy + w\log w)z^3}$$

$$t_1 = xy$$

$$t_2 = \log w$$

$$t_3 = wt_2 \qquad t_3' = w\log w$$

$$t_4 = t_1 + t_3$$

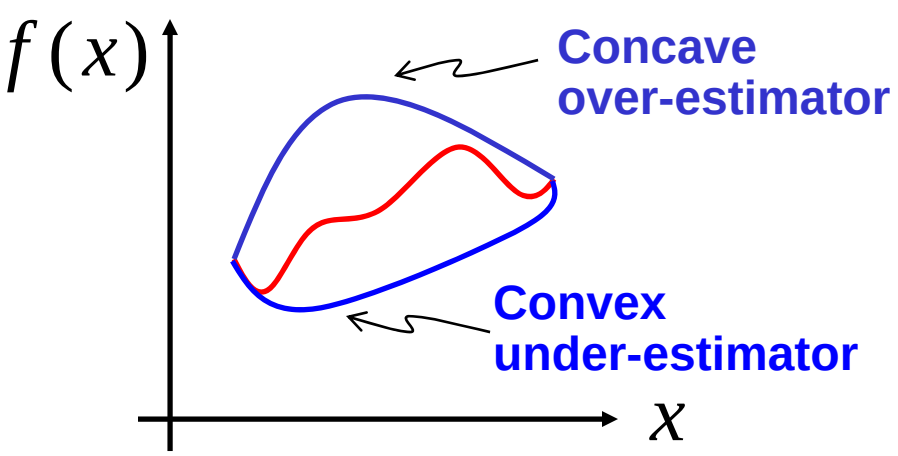
$$t_5 = \exp(t_4) \qquad t_5' = \exp(t_1 + t_3)$$

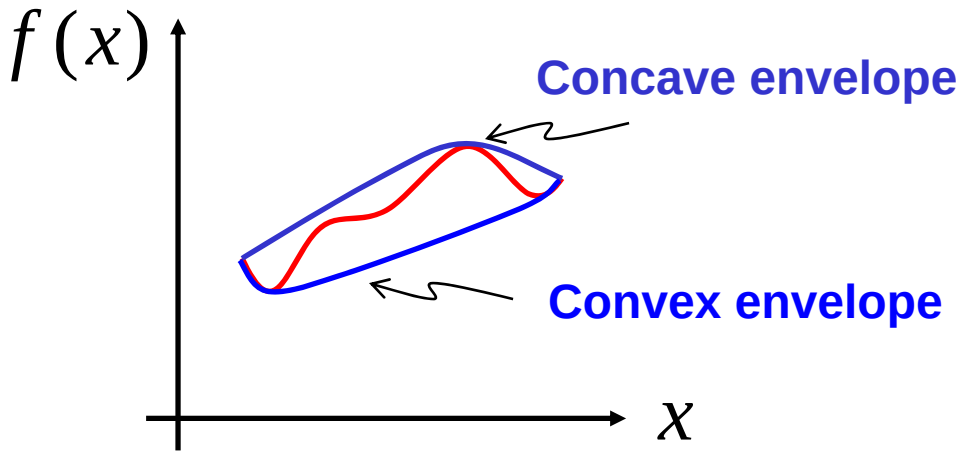
$$t_6 = z^3$$

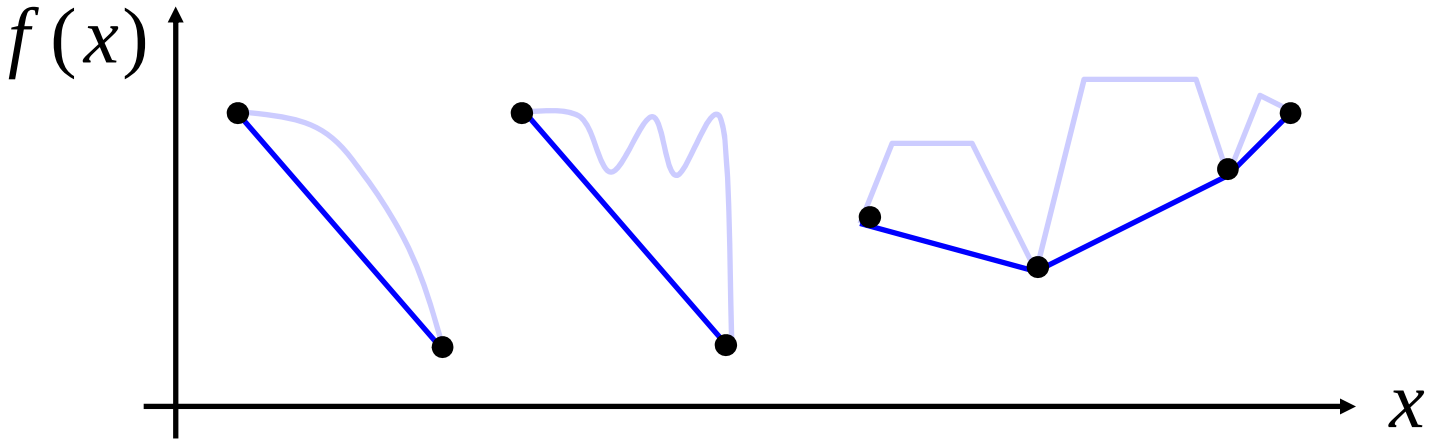
$$t_7 = t_5 t_6$$

$$t_8 = \sqrt{t_7} \qquad t_8' = \sqrt{t_5 t_6}$$

TIGHT RELAXATIONS







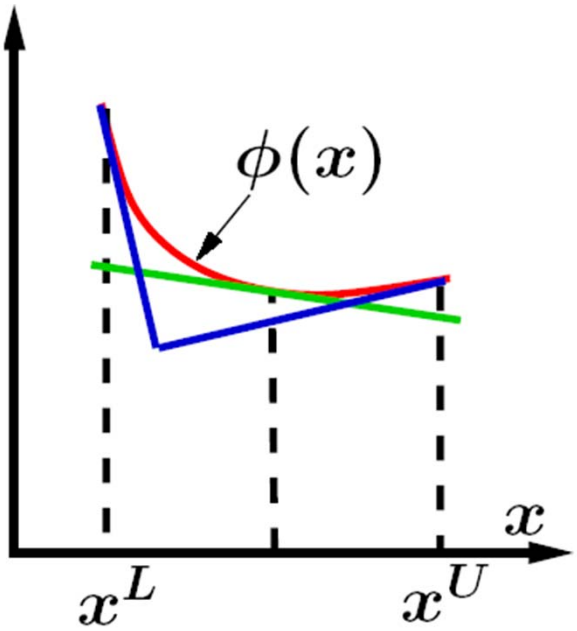
Convex/concave envelopes often finitely generated

(Tawarmalani and Sahinidis, 2001; Khajavirad and Sahinidis, 2013)

POLYHEDRAL OUTER APPROXIMATION

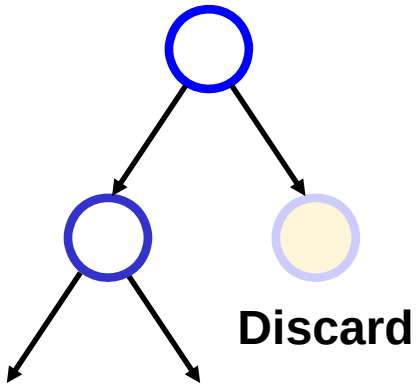
(Tawarmalani and Sahinidis, 2001, 2004)

- **Convex NLP solvers are not as robust as LP solvers**
- **Linear programs can be solved efficiently**
- **Outer-approximate convex relaxation by polyhedron**

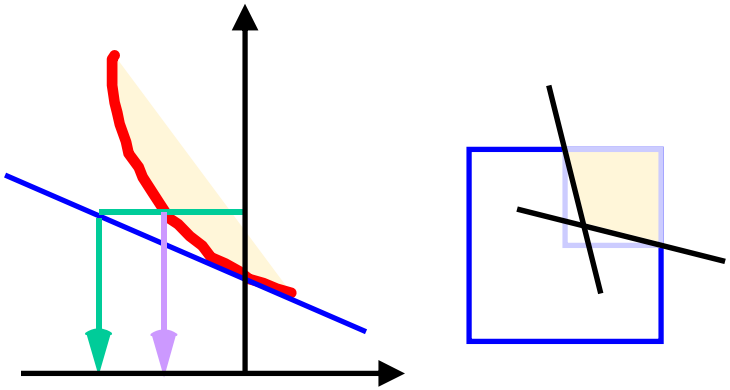


BRANCH-AND-REDUCE

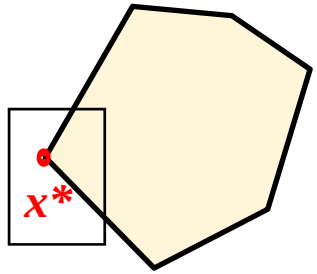
Search Tree



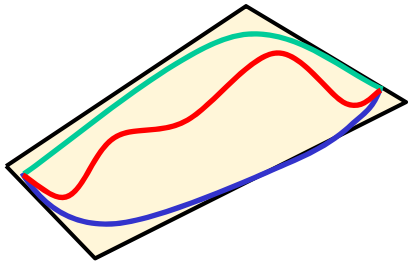
Range Reduction



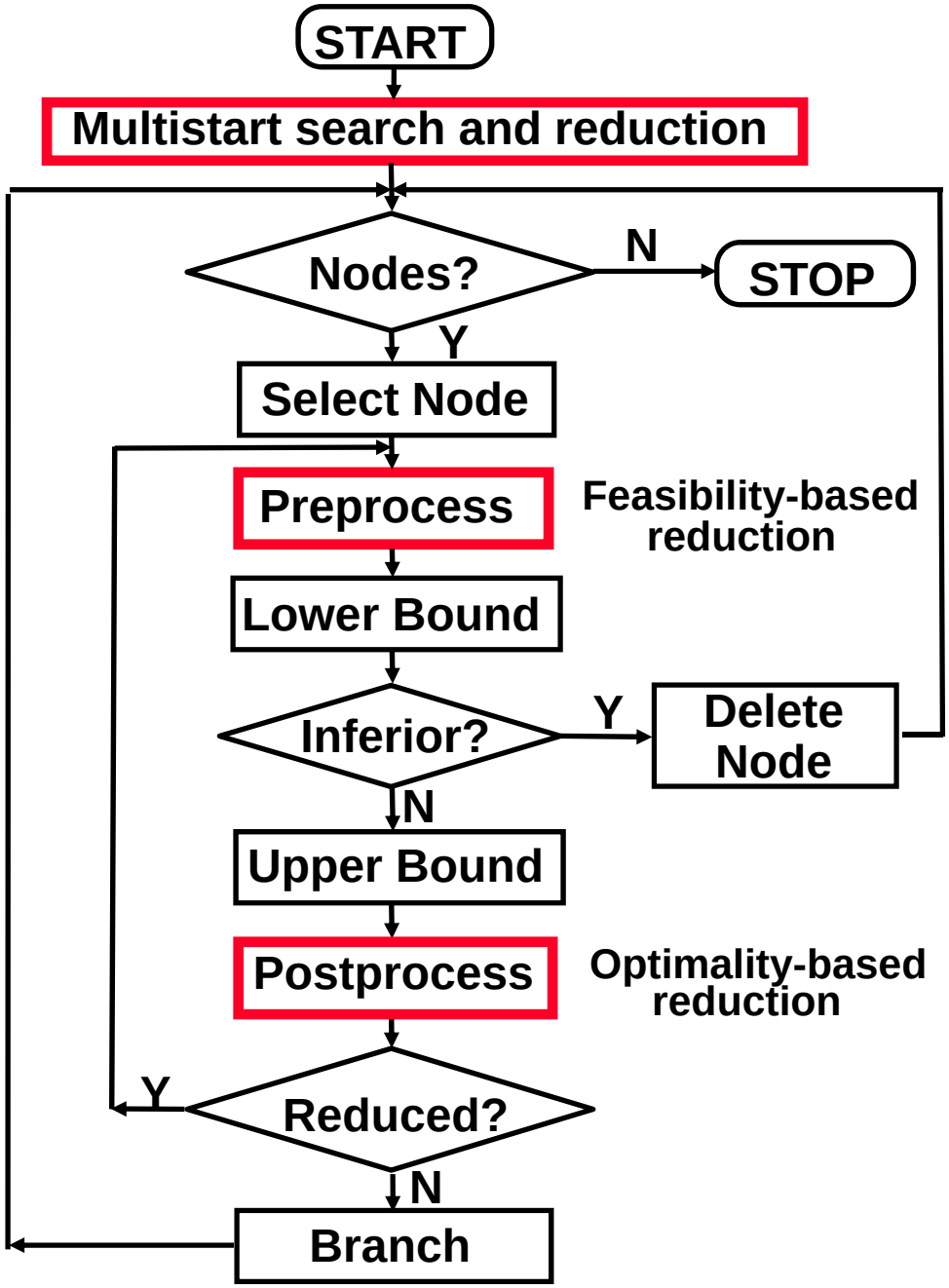
Finiteness



Convexification



BRANCH-AND-REDUCE



Branch-And-Reduce Optimization Navigator

Components

- Modeling language
- Preprocessor
- I/O handler
- Range reduction
- Interval arithmetic
- Sparse matrix routines
- Automatic differentiator
- Debugging facilities

Subsolvers

- NLP
  - MINOS, SNOPT, CONOPT, IPOPT
- LP
  - CPLEX, XPRESS, CLP

Availability

- Under GAMS and AIMMS
- On NEOS server
- Under MATLAB and YALMIP

BARON HISTORY

1991-93	Duality-based range reduction
	Constraint propagation
1994-95	Branch-and-bound system
	Finite algorithm for separable concave minimization
1996-97	Parser for factorable programs; nonlinear relaxations
	Links to MINOS and OSL
1997-98	Polyhedral relaxations; Link to CPLEX
	Compressed data storage, tree traversal, …
2002	Under GAMS
2004	Branch-and-cut
2005-07	Local search; memory management, …
2009	Multi-term envelopes
2010-13	Multi-variate and multi-constraint envelopes/relaxations
	Links to CLP and IPOPT
	Hybrid LP/NLP relaxations
2014	Irreducible Inconsistent Set for infeasible problems
	Under MATLAB and YALMIP

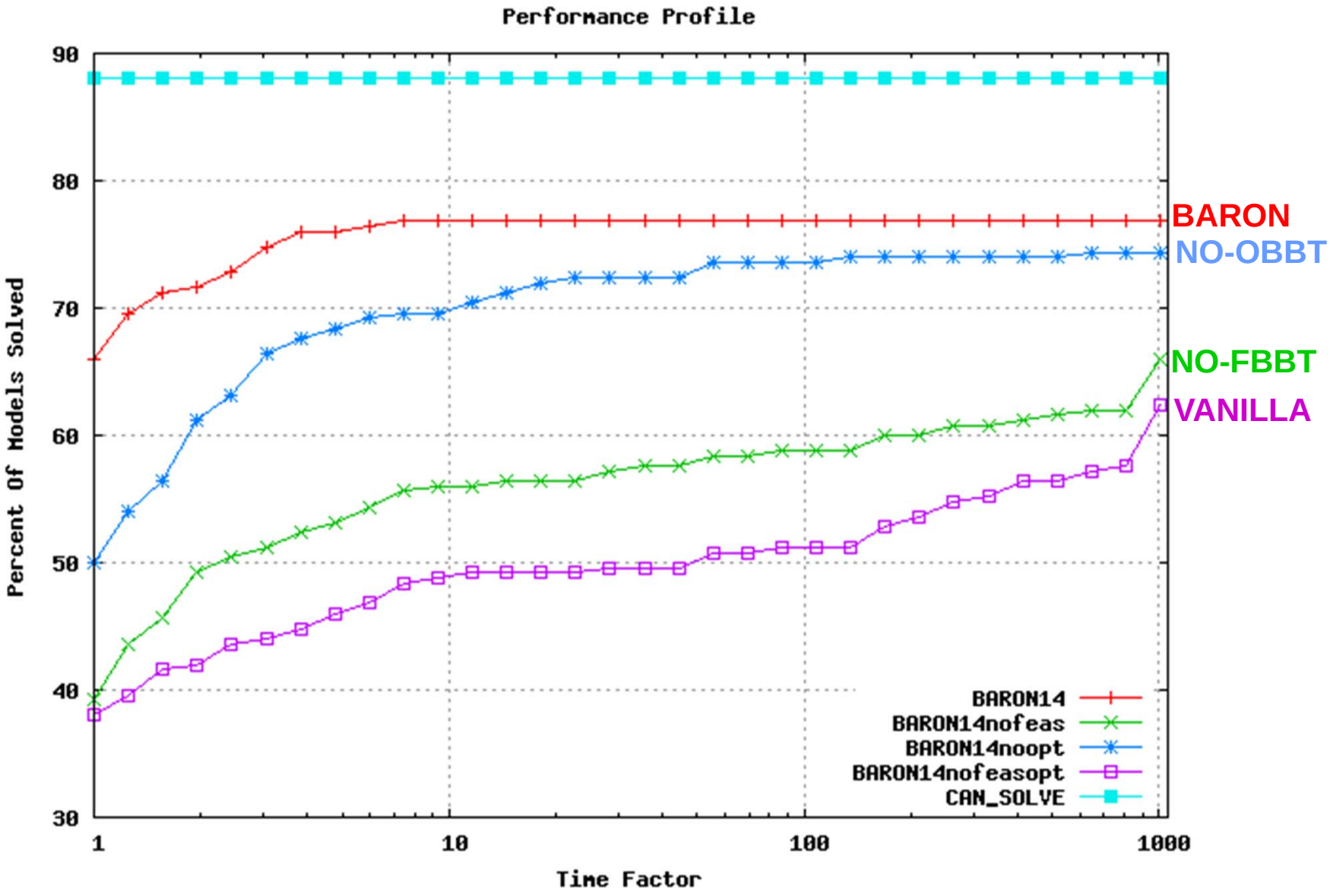
COMPUTATIONAL SETUP

- 64-bit Intel Xeon X5650 2.66Ghz processors
  - 12 processors sharing 48 GB RAM
  - Each BARON run restricted to single processor
  - BARON v. 14.0 (with CPLEX for LPs and MINOS-SNOPT-CONOPT-IPOPT for NLPs)
- All experiments under GAMS
  - 500 CPU seconds limit; optca=optcr=1e-6
  - Performance profiles with GAMS performance tool
  - Profiles based on best feasible solution (not gaps)
  - GAMS Examiner used to validate all solutions

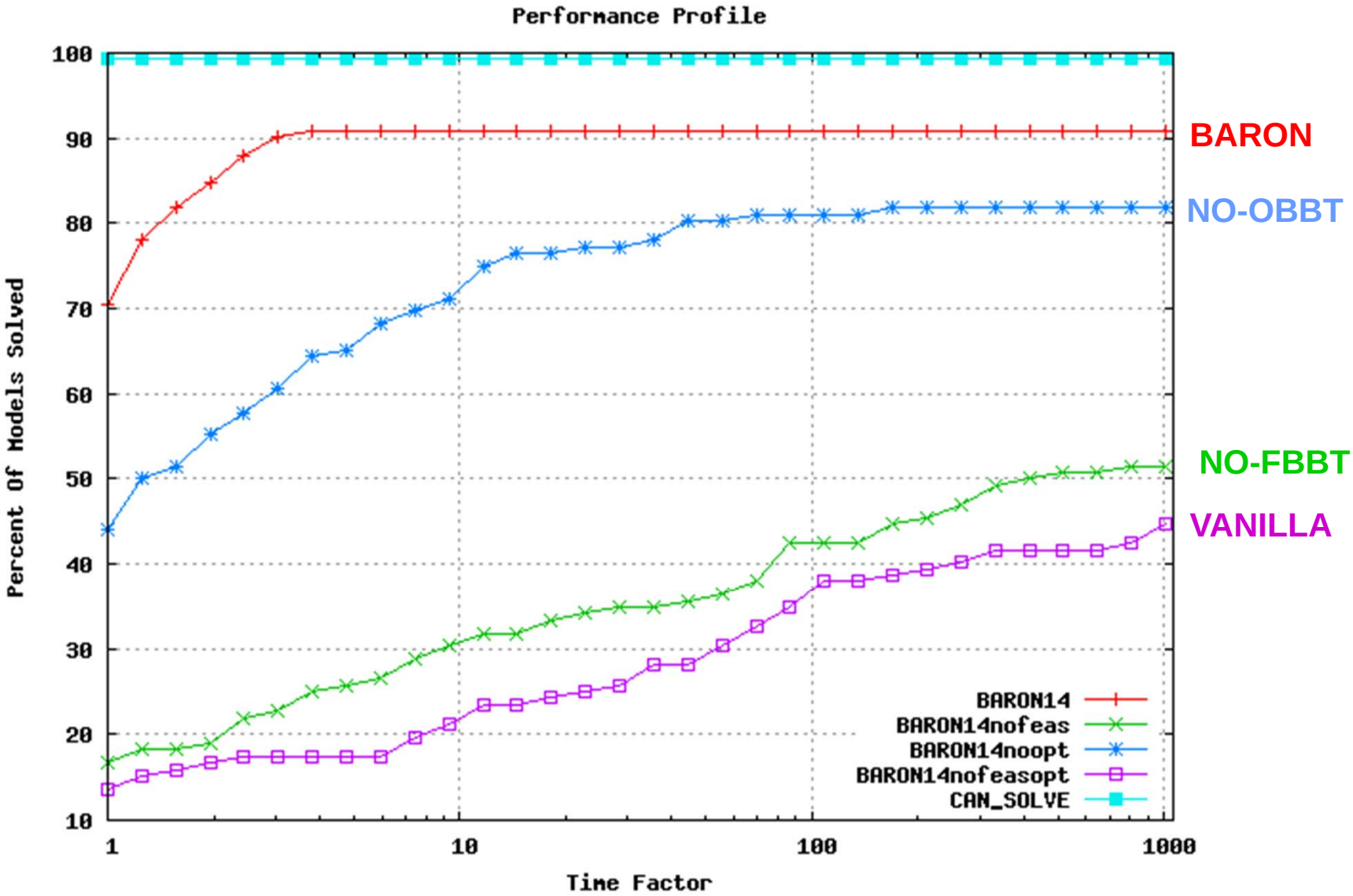
TEST SET

- **MINLPLIB**
  - **http://www.gamsworld.org/minlp**
  - **250 problems**
  - **Constraints: 64 (1—24972)**
  - **Variables: 92 (1—23827)**
  - **Discrete Variables: 30 (1—10920)**
- **IBMLIB**
  - **Developed under CMU-IBM open source MINLP project**
  - **132 problems**
  - **http://egon.cheme.cmu.edu/ibm/page.htm**
  - **Constraints: 606 (29—4981)**
  - **Variables: 344 (62—2721)**
  - **Discrete Variables: 92 (5—576)**

MINLPLIB (BOUND TIGHTENING)



IBMLIB (BOUND TIGHTENING)



BOUND TIGHTENING METHODS

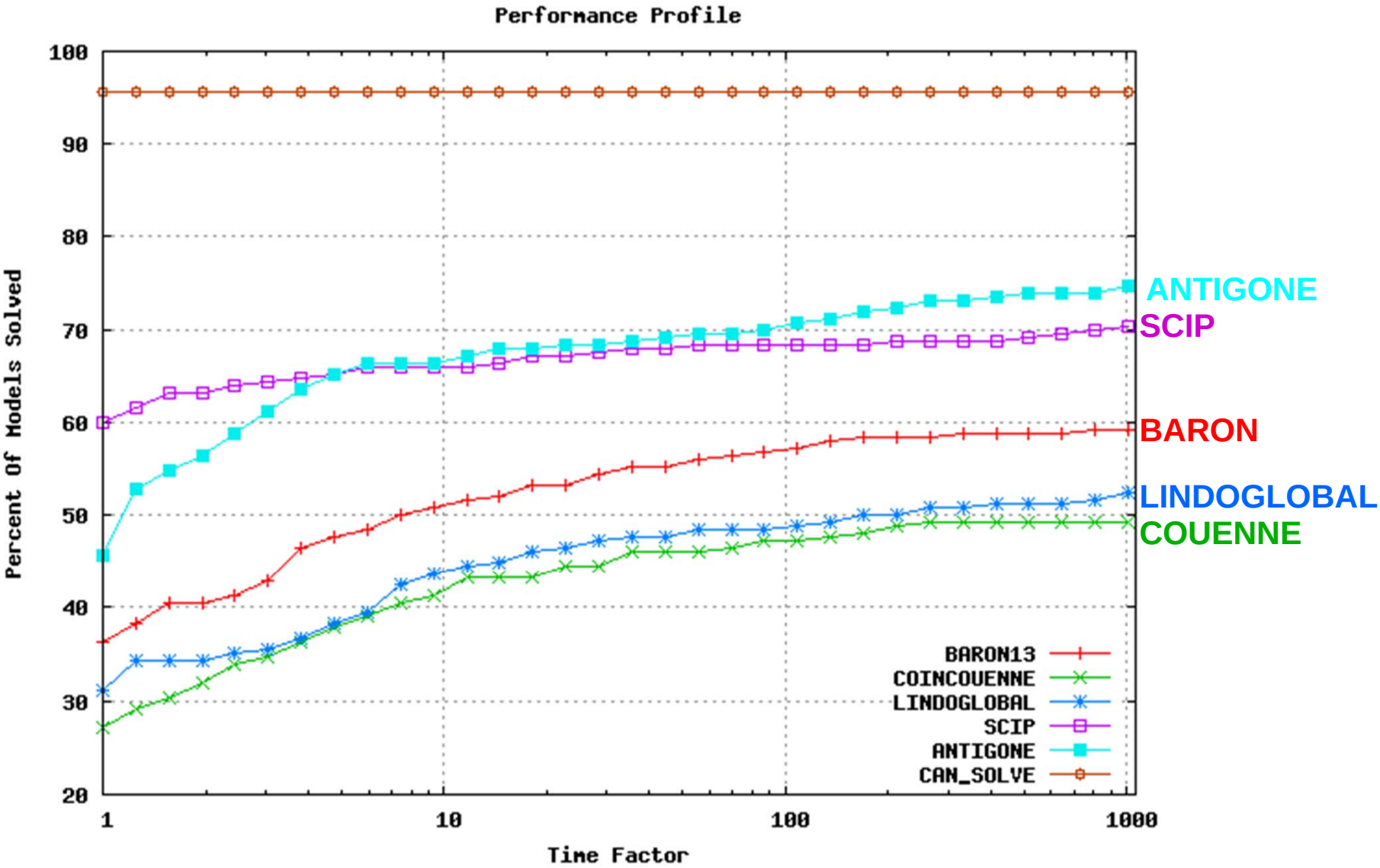
MINLPLIB	BARON	BARON NO FBBT	BARON NO OBBT	BARON VANILLA
# solved	148	95	134	74
# timeout	91	144	105	166
# failed	11	11	11	10
Time(sec)	56	115	69	162

IBMLIB	BARON	BARON NO FBBT	BARON NO OBBT	BARON VANILLA
# solved	82	21	58	18
# timeout	47	108	66	112
# failed	3	3	8	2
Time(sec)	84	312	144	353

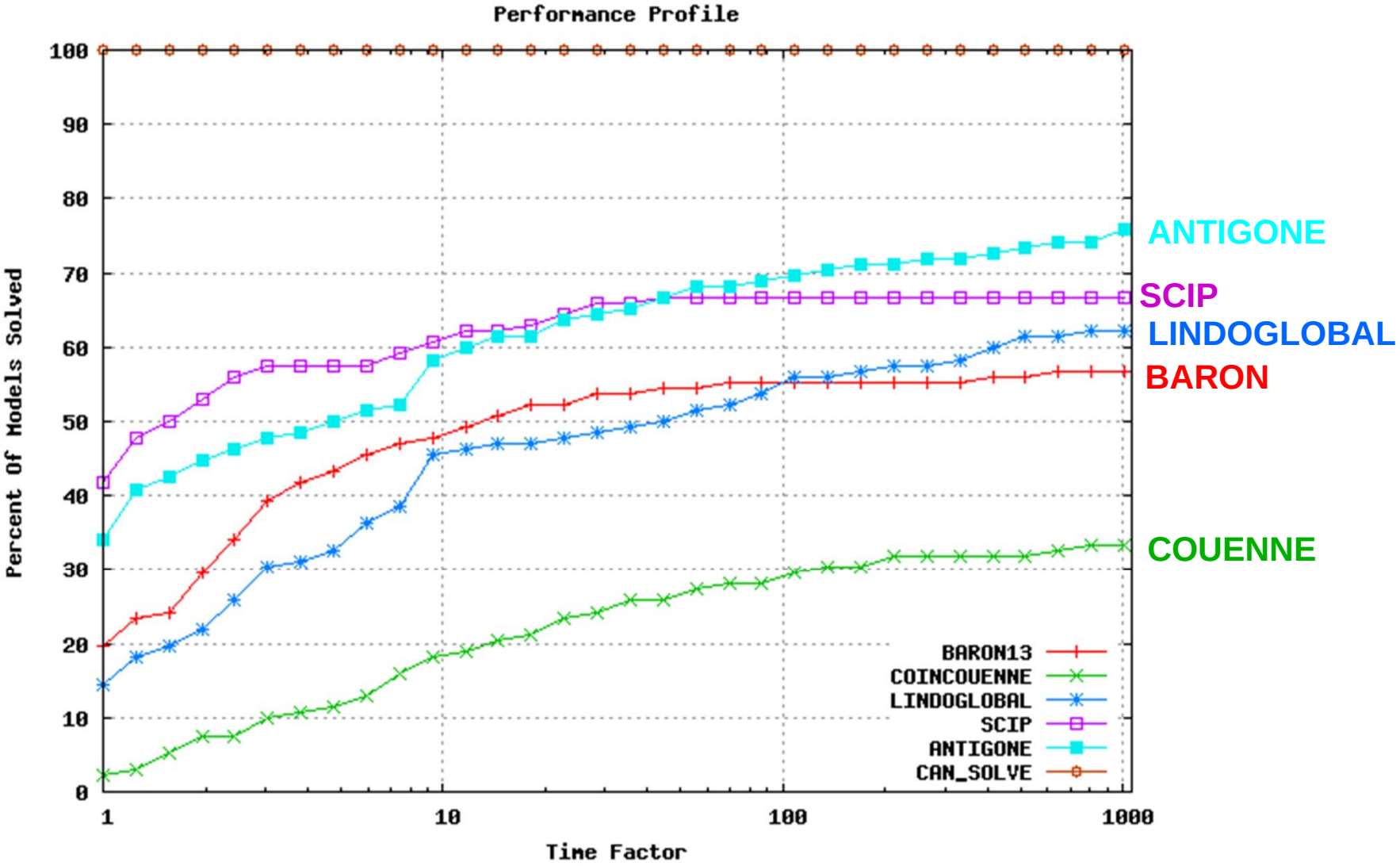
COMPARISON TO OTHER GLOBAL SOLVERS

- **BARON 13.0**
- **Antigone 1.1**
- **Couenne 0.4**
- **LindoGlobal 8.0**
- **SCIP 3.0**

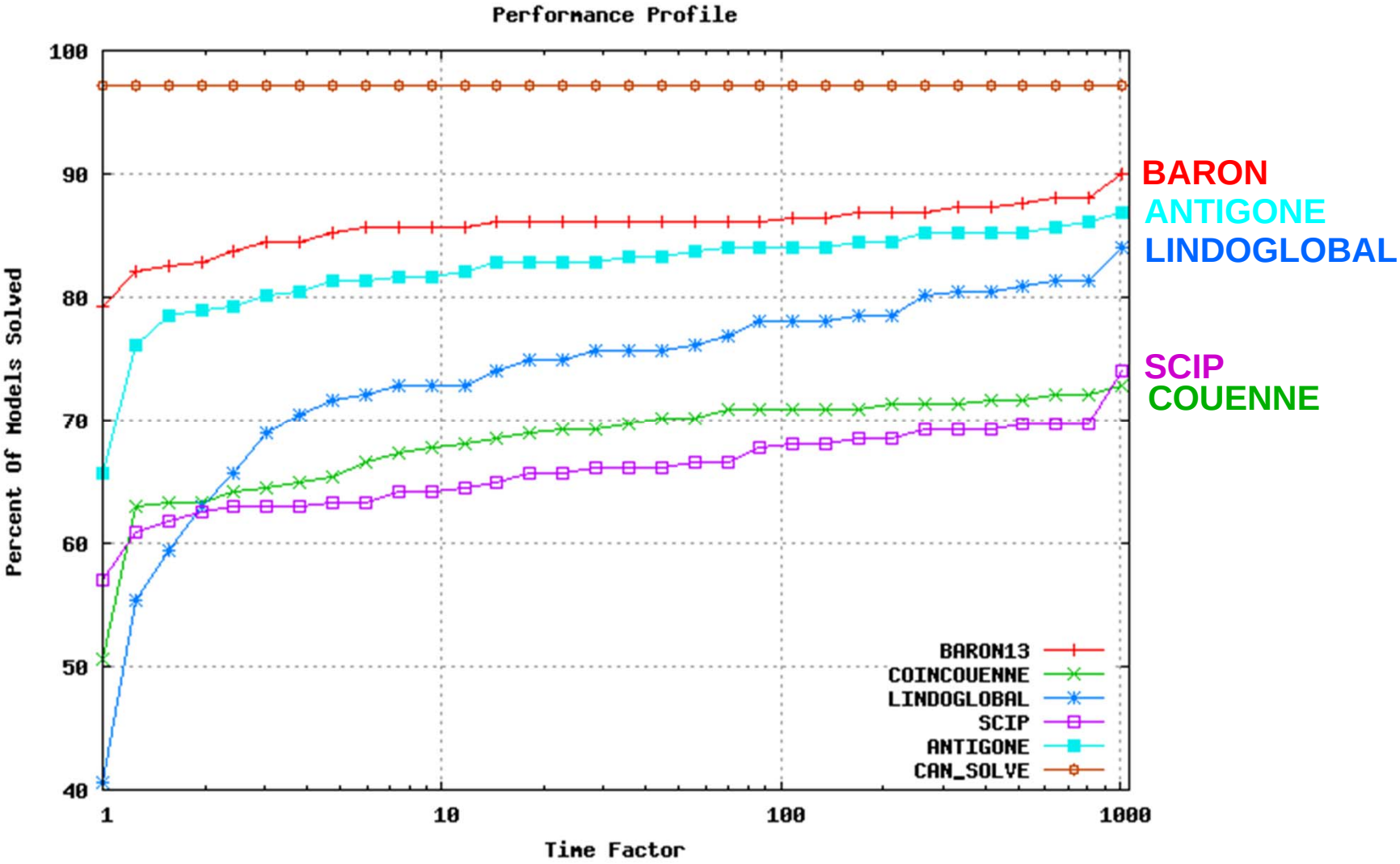
MINLPLIB (BARON13)



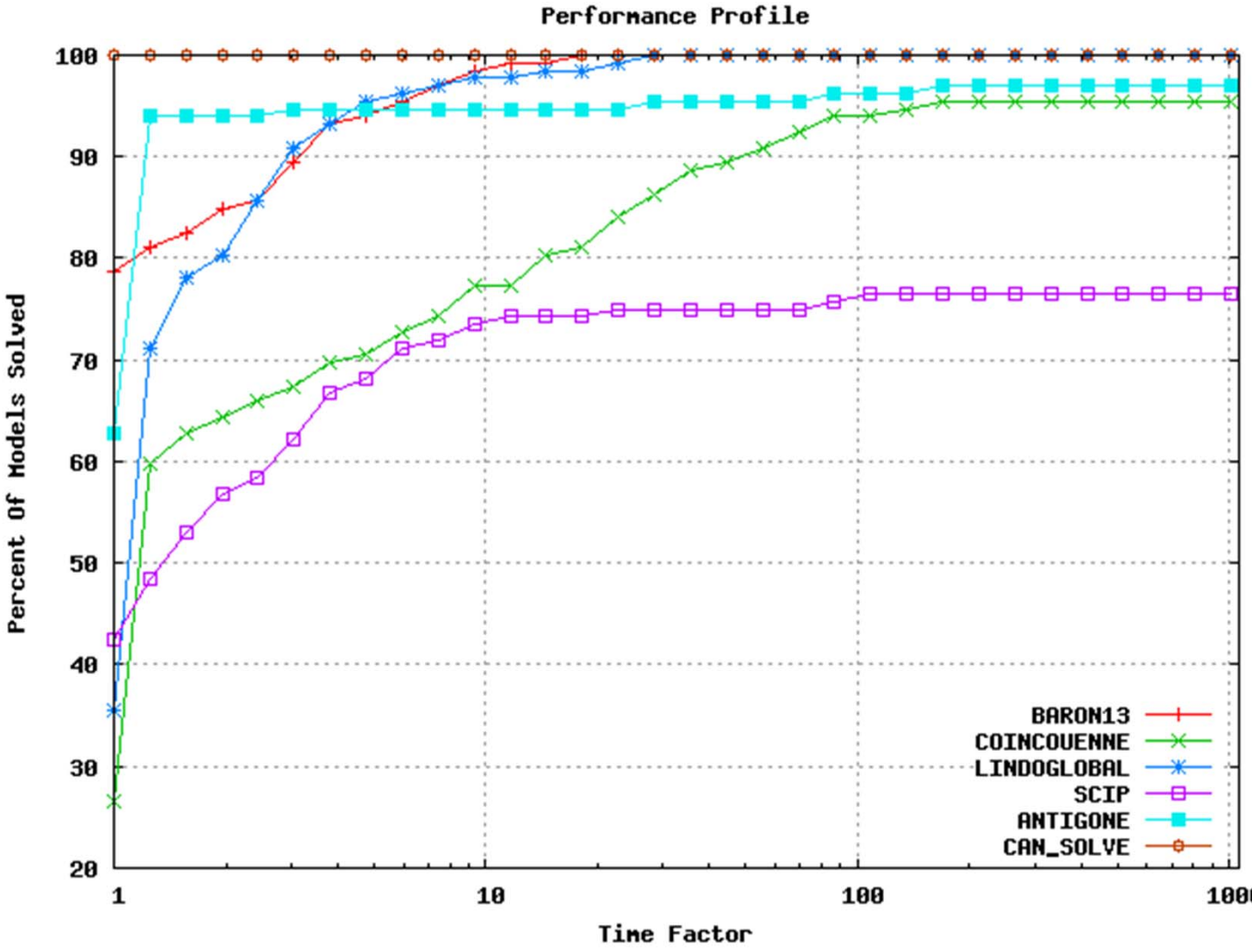
IBMLIB (BARON13)



relaxed-MINLPLIB (BARON13)



relaxed-IBMLIB (BARON13)



BARON
LINDOGLOBAL
ANTIGONE
COUENNE

SCIP

BARON 14.0

- **Constraint classification**
  - **Knapsack**
  - **GUB**
  - **VUB**
- **Probing**
  - **Coefficient reduction**
  - **Conflict graph**
  - **Implication graph**
- **Cuts**
  - **Knapsack cover cuts**
  - **Knapsack cover cuts with GUB constraints**
  - **Clique cuts**
  - **Flow cover cuts**

BARON 14.0

- **LP/MIP relaxations**
  - **Rework LP interface**
  - **Feasibility/infeasibility checker**
  - **Row management**
  - **Reintroduce MIP relaxations**
    - » **Sahinidis and Tawarmalani**
      - **INFORMS 2008**
      - **ISMP 2009**

MIP RELAXATIONS

- **Better lower bound/Integer infeasibility**
  - **Pruning**
- **Integer feasible solutions**
  - **Local search heuristics**
- **Expensive compared to LP relaxation**
  - **Solved only if not pruned by LP relaxation**
  - **Do not solve to optimality**
  - **Dynamic strategy to decide when to solve MIP relaxations**

MIP RELAXATION PARAMETERS

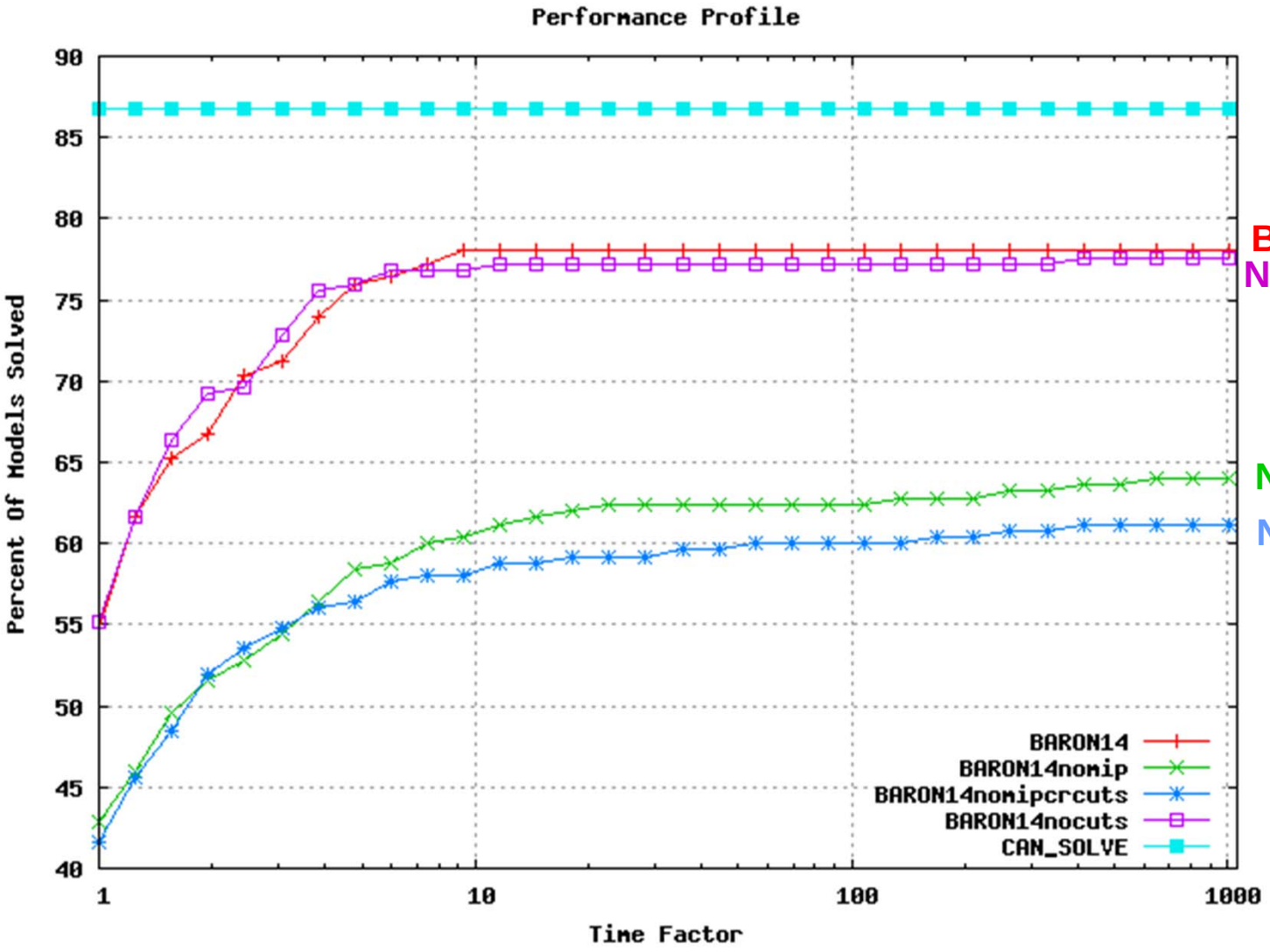
- **Relative optimality tolerance for MIP relaxation**
  - **Exact, epsr, 5%, 10%, 20%, …**
- **Absolute optimality tolerance for MIP relaxation**
  - **0, epsa, 10*epsa, …**
- **Limit on the maximum number of MIP relaxation nodes**
  - **1, 10, 100, …, or no limit**
- **Minimum of MIP relaxation nodes**
  - **1, 10, 100, …, or no min**
- **Emphasis on feasibility if no feasible solution**

BARON 13 vs. 14

	MINLPLIB		IBMLIB	
	BARON 13	BARON 14	BARON 13	BARON 14
# solved	135	148	66	82
# timeout	90	91	66	47
# failed	25	11	0	3
Time(sec)	68	56	107	84

	MINLPLIB		IBMLIB	
	BARON 13	BARON 14	BARON 13	BARON 14
LP time	37%	27%	41%	39%
NLP time	29%	13%	23%	10%
MIP time	-	45%	-	30%
Total	66%	85%	64%	79%

MINLPLIB (MIP TECHNIQUES)

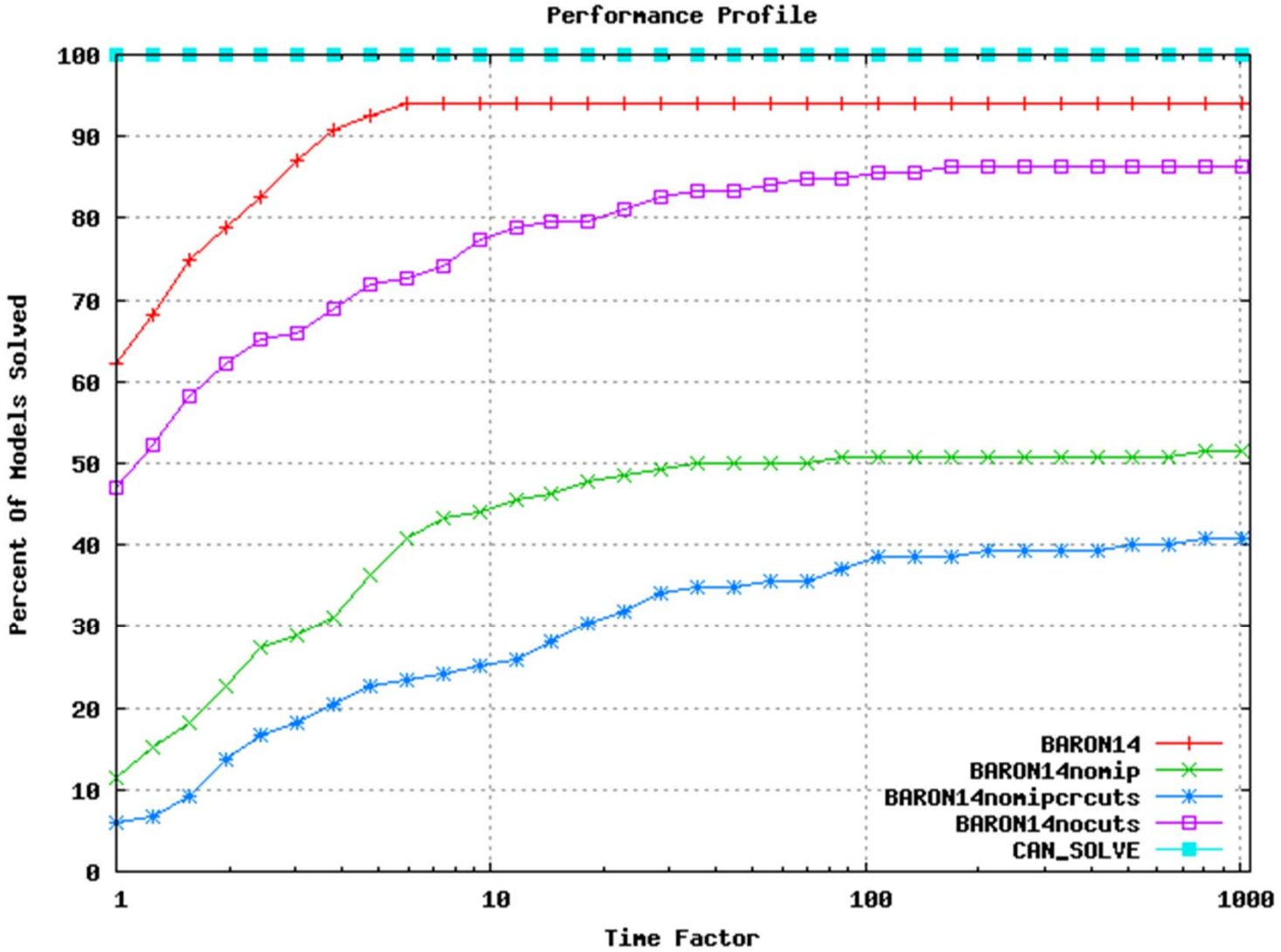


BARON
NO CUTS, CR

NO MIP

NO MIP, CUTS, CR

IBMLIB (MIP TECHNIQUES)



BARON

NO CUTS, CR

NO MIP

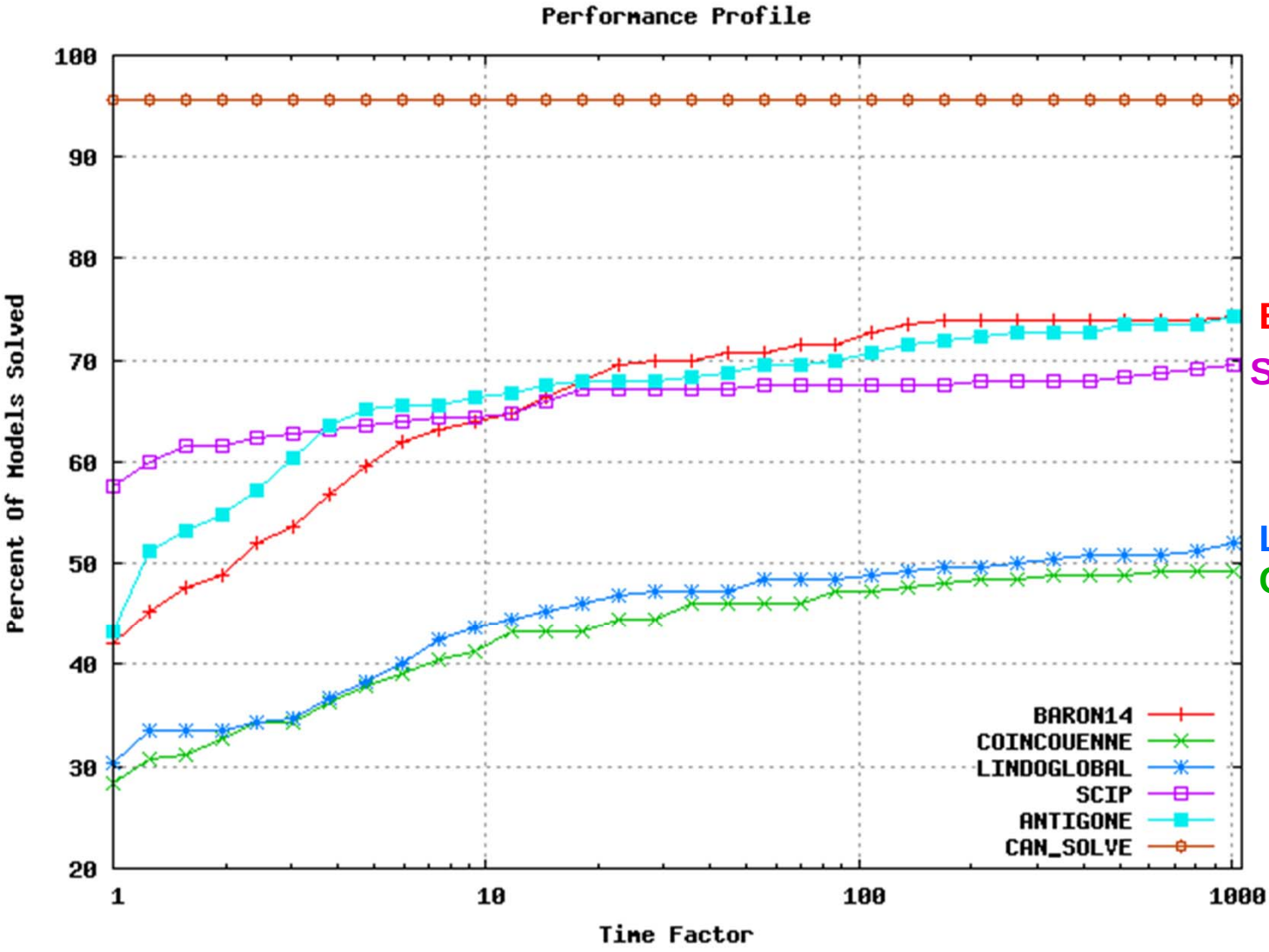
NO MIP, CUTS, CR

MIP TECHNIQUES

MINLPLIB	BARON	BARON NO MIP	BARON NO CR/Cuts	BARON NO MIP/CR/Cuts
# solved	148	137	144	132
# timeout	91	103	92	110
# failed	11	10	14	8
Time	56	63	57	67

IBMLIB	BARON	BARON NO MIP	BARON NO CR/Cuts	BARON NO MIP/CR/Cuts
# solved	82	63	66	47
# timeout	47	69	61	85
# failed	3	0	5	0
Time	84	126	121	190

MINLPLIB (BARON14)



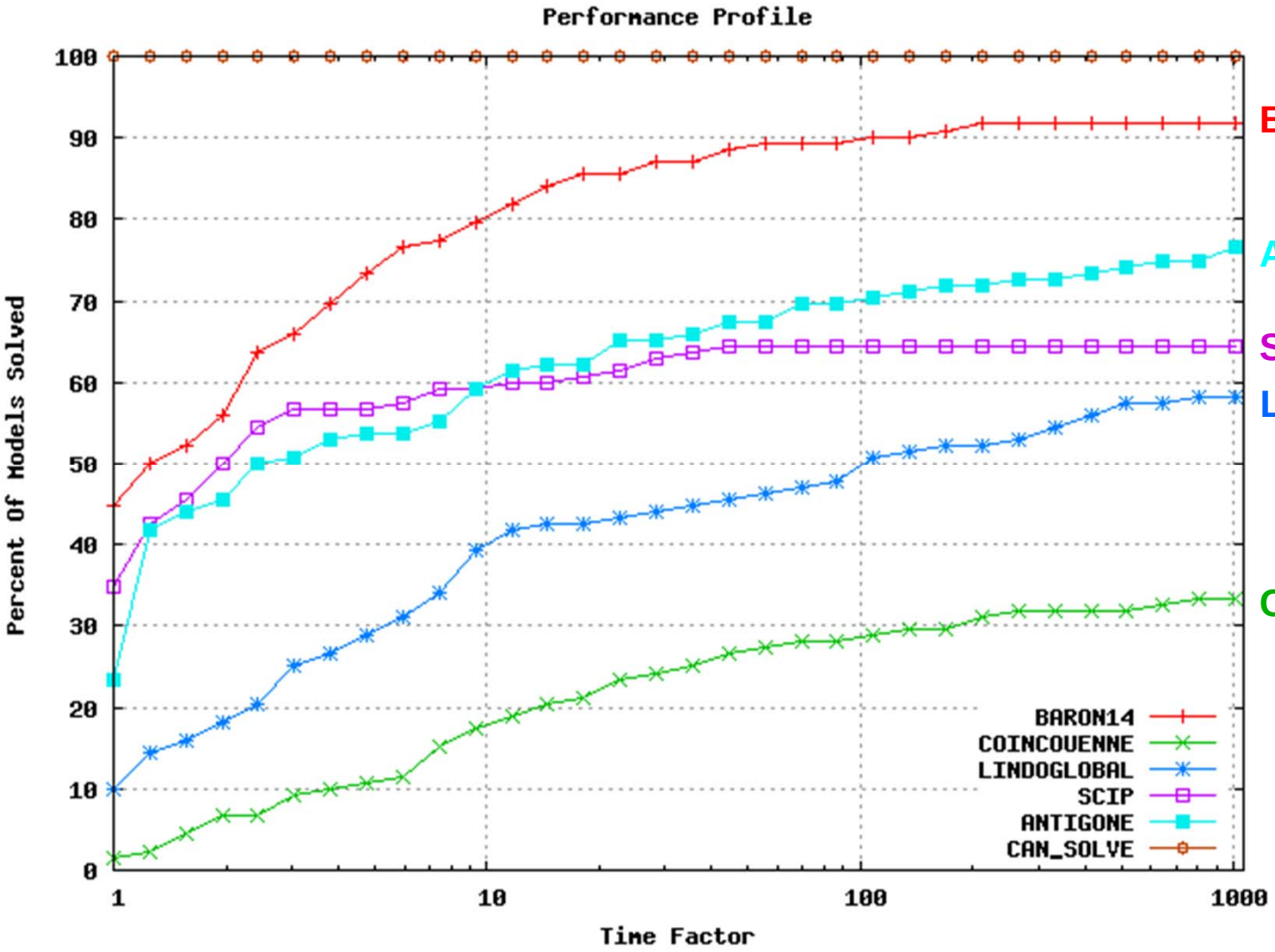
BARON **ANTIGONE**

SCIP

LINDOGLOBAL

COUENNE

IBMLIB (BARON14)



BARON

ANTIGONE

SCIP

LINDOGLOBAL

COUENNE

GLOBAL SOLVERS

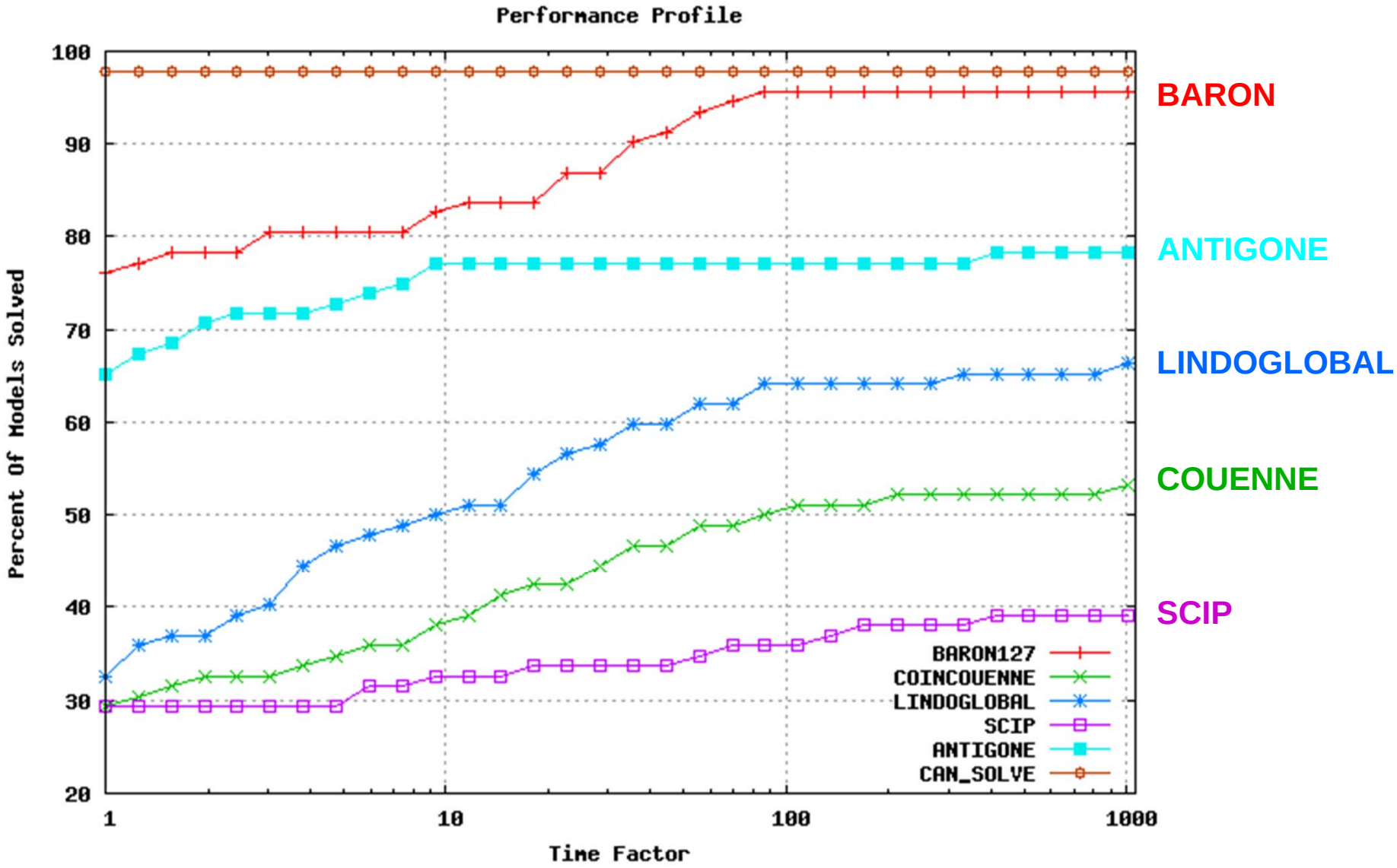
MINLPLIB	BARON	COUENNE	LINDOGLOBAL	SCIP	ANTIGONE
# solved	148	110	95	144	151
# timeout	91	86	104	87	86
# failed	11	54	51	19	13
Time(sec)	56	117	145	66	59

IBMLIB	BARON	COUENNE	LINDOGLOBAL	SCIP	ANTIGONE
# solved	82	39	47	79	59
# timeout	47	74	78	32	73
# failed	3	19	7	21	0
Time(sec)	84	278	206	93	144

THANK YOU!

92 MPECs FROM MPECLIB

(Zhang and Sahinidis, 2014)



Con: 512 (2—5699), Var: 510 (3—5671); Equil: 417 (1—4480)

26 PROBLEMS FROM globallib AND minlplib

(Tawarmalani and Sahinidis, 2005)

	Minimum	Maximum	Average
Constraints	2	513	76
Variables	4	1030	115
Discrete variables	0	432	63

EFFECT OF CUTTING PLANES

	Without cuts	With cuts	% reduction
Nodes	23,031,434	253,754	99
Nodes in memory	622,339	13,772	98
CPU sec	275,163	20,430	93

CONVEXITY EXPLOITATION IN MINLPLIB

(Khajavirad and Sahinidis, 2014)