Solving Convex MINLPs with MINOTAUR: Presolving, Cuts and More

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About this talk

The Team

About this talk

The Agenda

1. Describe the Minotaur framework
2. Glimpse of what is under the hood
3. Two main themes:
   - Exploit specific problem structures
   - Customize specific components of Branch-and-Cut
4. Illustrate how you can extend or customize it

What is Minotaur?

Mixed
Integer
Nonlinear
Optimization
Toolkit:
Algorithms,
Underestimators,
Relaxations.

http://wiki.mcs.anl.gov/minotaur

- Completely open-source: BSD License
- Source code, libraries, binaries available
- Well tested on Linux, Mac
- Documentation on the wiki, examples in the source

Three ways to use Minotaur

1. Use the binaries (through AMPL or Pyomo)
    - NLP based branch-and-bound
    - LP-NLP based branch-and-bound
    - QP-diving
    - Solvers for nonconvex problems under development
    - Available as minotaur-xxx-bin-yyy.tar.gz

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   - Use existing interfaces, methods, engines, branchers, tree-manager, ...
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3. Build your own
   - Modify/add your own code to Minotaur
   - Available as minotaur-xxx-src.tar.gz
   - C++, modular

Inside Minotaur: Three Main Components

Core

1. Problem Description Classes
   - Function,
   - NonlinearFunction, LinearFunction,
   - Variable, Constraint, Objective, etc.

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Engines

1. OSI-LP (coin-or.org)
   - CLP
   - CPLEX
   - GUROBI
2. BQPD
3. qpOASES
4. IPOPT
5. Filter-SQP

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Interfaces

1. AMPL
2. C++

Nonlinear Functions in Minotaur

Base Class: NonlinearFunction

```
virtual void computeBounds(...);
virtual Double eval(...);
virtual void evalGradient(...);
virtual void evalHessian(...);
...
```

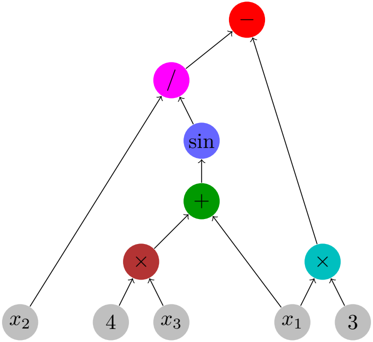
Derived Classes:

- AMPLNonlinearFunction
  Queries AMPL's ASL library for the above functions

- CGraph
  Nonlinear "factorable" function is stored as a computational graph

Computational Graph

Consider a function $f : \mathbb{R}^3 \to \mathbb{R}, \quad f = \frac{x_2}{\sin(4 \times x_3 + x_1)} - 3 \times x_1$



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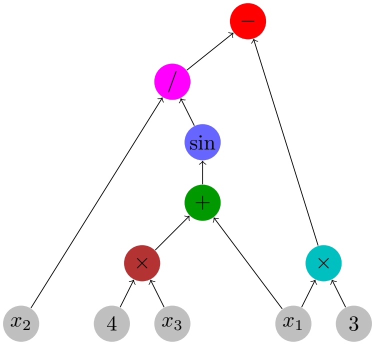


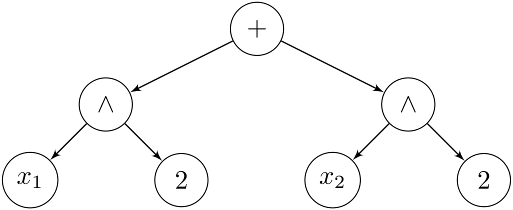
Minotaur's computational graph of f allows us to:

- evaluate f at a given point,
- obtain bounds, under-estimators and over-estimators of f,
- evaluate "exact" gradient and hessian at a given point "cheaply" (Automatic Differentiation).

It is the default class for storing nonlinear functions.

Operators Supported: $+, -, \times, /, log, ln, a^x, x^a, sin, cos, tan, sinh, sin^{-1}, \ldots$

Easy to Construct Them



Suppose we have a function $x_1^2 + x_2^2$

Easy to Construct Them

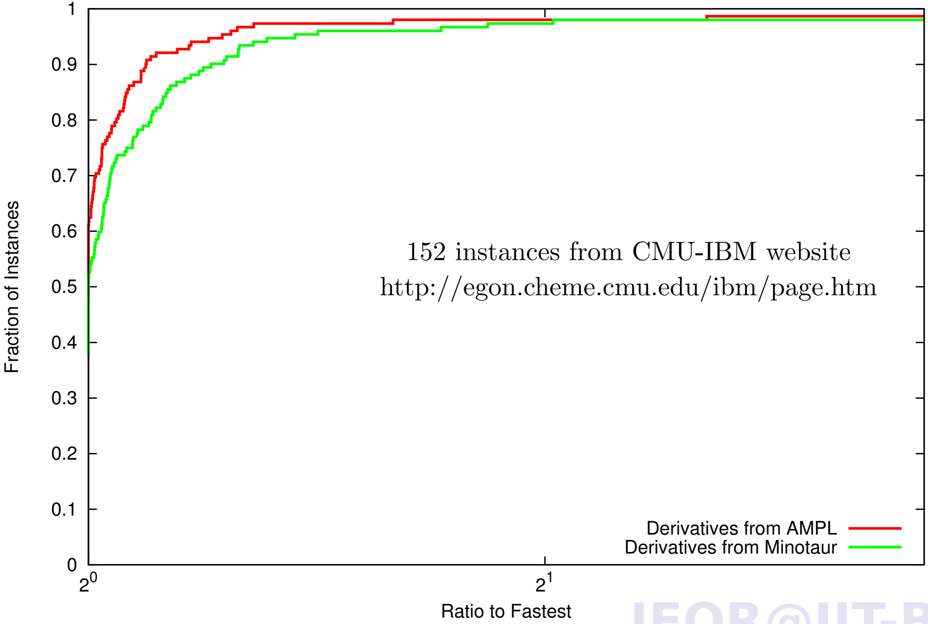
```
ProblemPtr p = (ProblemPtr) new Problem();
VariablePtr x1 = p->newVariable(0, 1, Binary, "x1");
VariablePtr x2 = p->newVariable(0, 1, Binary, "x2");

CGraphPtr cg = (CGraphPtr) new CGraph();
CNode *n2 = cg->newNode(2.0);
CNode *nx1 = cg->newNode(x1);
CNode *nx2 = cg->newNode(x2);

CNode *np1 = cg->newNode(OpPow, nx1, n2);
CNode *np2 = cg->newNode(OpPow, nx2, n2);
n2  = cg->newNode(OpPlus, np1, np2);

cg->setOut(n2);
cg->finalize(); cg->write(std::cout);
```

Time per Node in Branch-and-Bound



Exploiting Structure Through Handlers

Where can we exploit problem structure?

- Relaxing
- Bounding
- Checking Feasibility
- Separating
- Branching
- Presolving

Methods in Handler Class

```
relaxNodeFull()
relaxNodeInc()
presolve()
presolveNode()
isFeasible()
separate()
getBranchingCandidates()
branch()
```

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`presolveNode()`
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`getBranchingCandidates()`
`branch()`

Example 1: NLP Based Branch-and-Bound for Convex MINLPs

- We need a handler for `isFeasible()`, `getBranchingCandidates()` and `branch()` only
- IntVarHandler does all three

A Very Simple Branch-and-Bound Solver

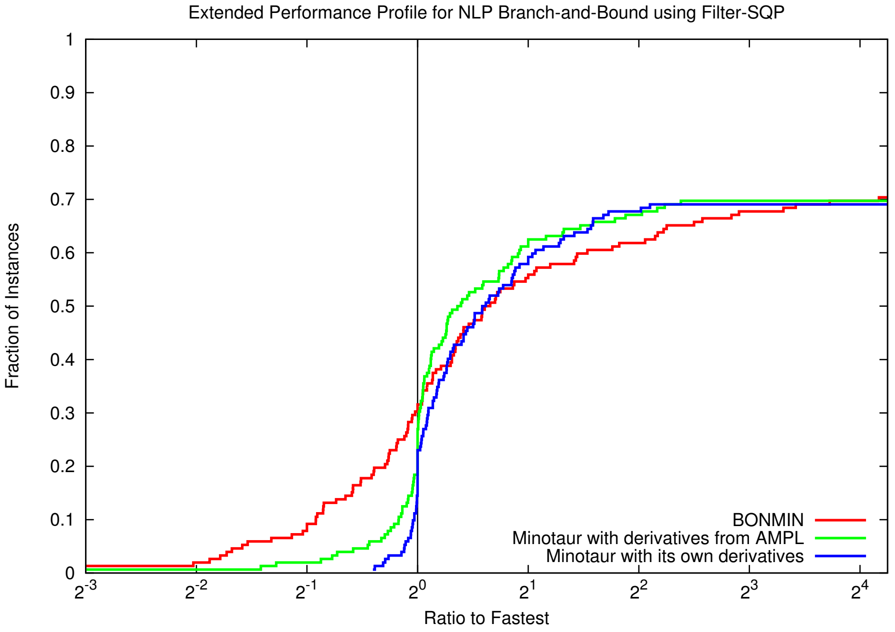
```
BranchAndBound *bab = new BranchAndBound(env, p);

v_hand = new IntVarHandler(env, p);
handlers.push_back(v_hand);

e = new FilterSQPEngine(env);
rel_br = new ReliabilityBrancher(env, handlers);
rel_br->setEngine(e);

nproc = new BndProcessor(env, e, handlers);
nproc->setBrancher(rel_br);
bab->setNodeProcessor(nproc);

nr = new NodeIncRelaxer(env, handlers);
nr->setEngine(e);
bab->setNodeRelaxer(nr);
bab->solve();
```



Extended Performance Profile for NLP Branch-and-Bound using Filter-SQP
Fraction of Instances
Ratio to Fastest
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2^{-3}
2^{-2}
2^{-1}
2^{0}
2^{1}
2^{2}
2^{3}
2^{4}
BONMIN
Minotaur with derivatives from AMPL
Minotaur with its own derivatives

Enhancing Branch-and-Bound

Example 2: NLP Based Branch-and-Bound for Convex MINLPs with Pre-solving

- IntVarHandler for `isFeasible()`, `getBranchingCandidates()` and `branch()`
- LinearHandler and NlPresHandler for `presolve()`

Enhancing Branch-and-Bound

Example 2: NLP Based Branch-and-Bound for Convex MINLPs with Presolving

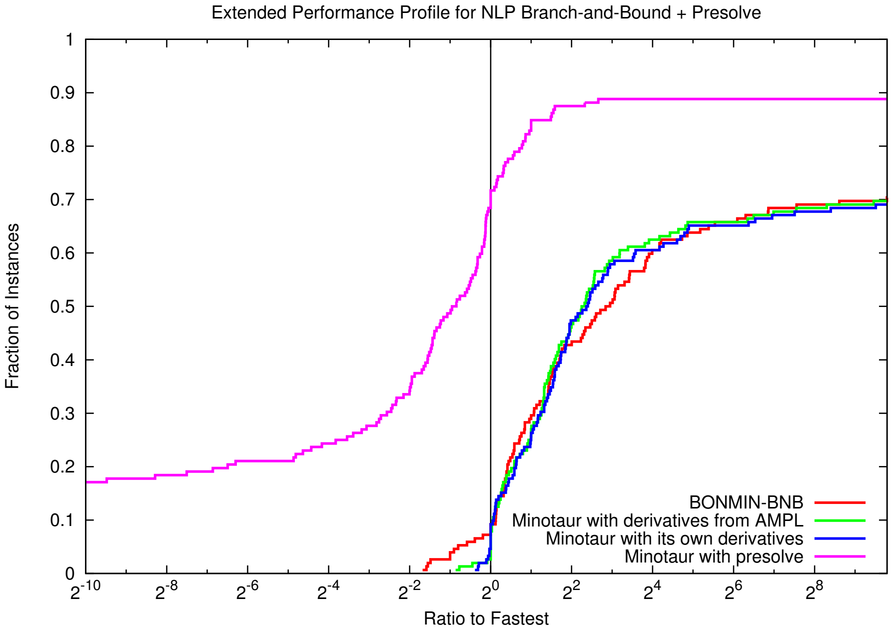
- IntVarHandler for `isFeasible()`, `getBranchingCandidates()` and `branch()`
- LinearHandler and NlPresHandler for `presolve()`

Basic functions in presolve:

- Tighten bounds on variables and constraints.
- Fix/remove variables.
- Identify and remove redundant constraints.
- Check duplicacy.

Advanced functions in presolve:

- Improve coefficients.
- Derive implications and conflicts.
- Quadratic binary to linear


Extended Performance Profile for NLP Branch-and-Bound + Presolve
Fraction of Instances
Ratio to Fastest
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2^-10
2^-8
2^-6
2^-4
2^-2
2^0
2^2
2^4
2^6
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BONMIN-BNB
Minotaur with derivatives from AMPL
Minotaur with its own derivatives
Minotaur with presolve

Pϵrspϵctive Rϵformulation

Recall Nick Sawaya's talk yesterday

If $g(x) \leq 0$ is a constraint in a MINLP, and the single variable y forces all x to zero, and $g(0) = 0$, then the constraint can be replaced by
$yg\left(\frac{x}{y}\right) \leq 0$

Perspective Reformulation

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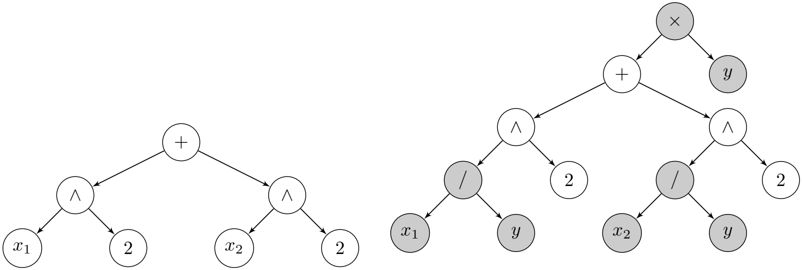

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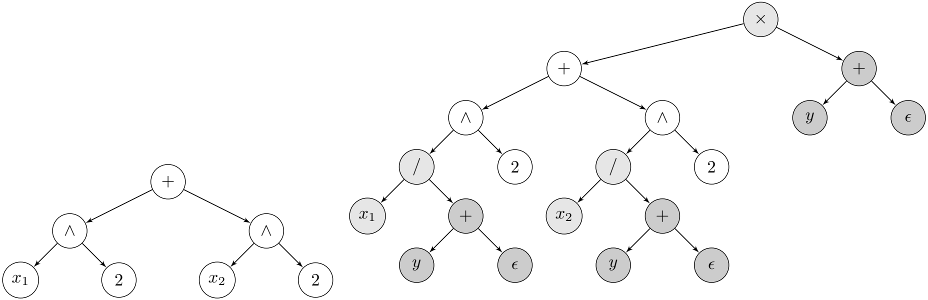
$$yg\left(\frac{x}{y}\right) \leq 0 \quad \rightarrow \quad (y+\epsilon)\, g\left(\frac{x}{y+\epsilon}\right) \leq 0$$

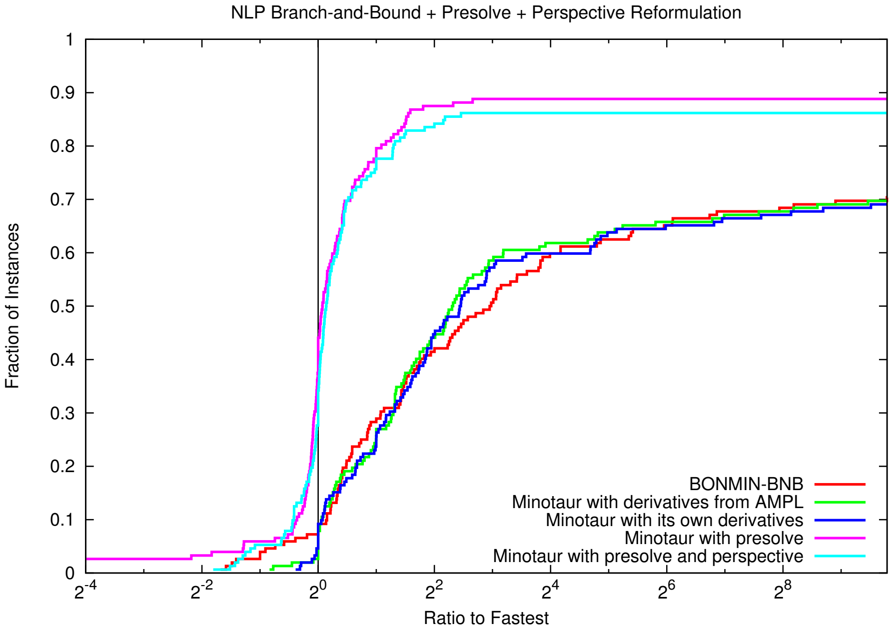


Easy to Implement (≈ 70 lines)

```
ynode = cg->newNode(y);
anode = cg->newNode(eps);
ynode = cg->newNode(OpPlus, anode, ynode);
// visit all nodes that have variables in them
for (it = cg->vars_.begin(); it!=cg->vars_.end();
     ++it) {
  v = *it;
  if (v != z) {
    mit = cg->varNode_.find(v);
    dnode = mit->second;
    cg->varNode_.erase(mit);

    vnode = cg->newNode(v);
    anode = cg->newNode(OpDiv, vnode, ynode);

    // set parents of anode.
    ...
```


NLP Branch-and-Bound + Presolve + Perspective Reformulation
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Ratio to Fastest
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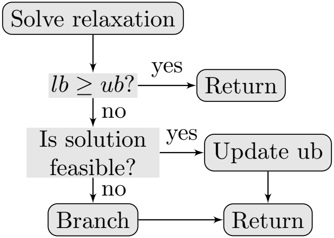
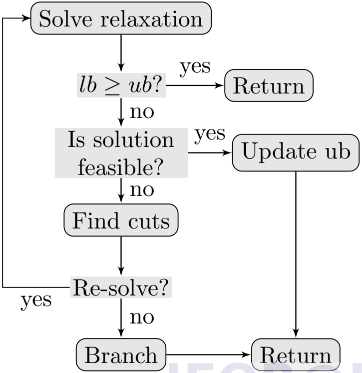
Minotaur with its own derivatives
Minotaur with presolve
Minotaur with presolve and perspective

Example 3: LP/NLP Based Branch-and-Bound (QG)

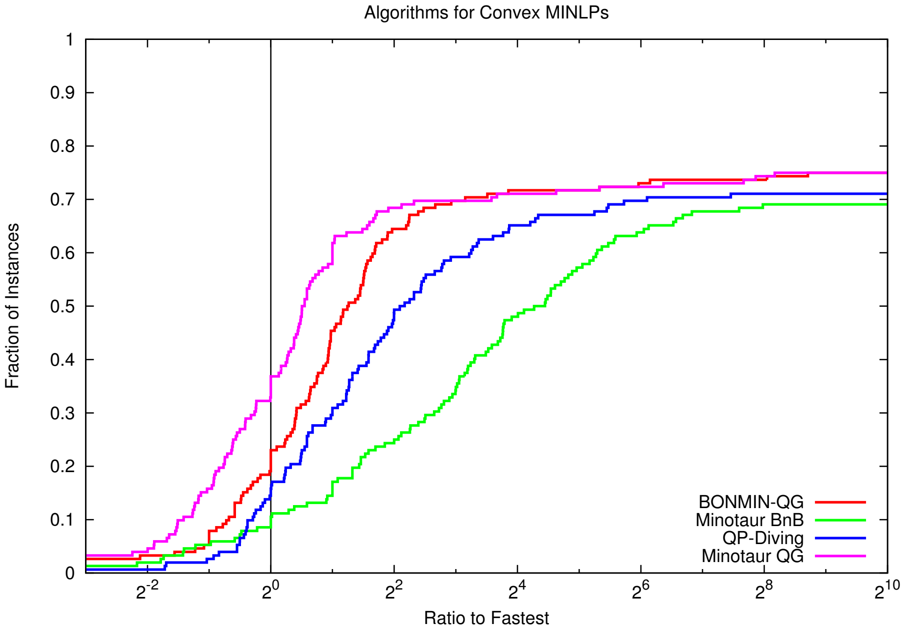
- IntVarHandler for `isFeasible()`, `getBranchingCandidates()` and `branch()`
- QGHandler for `isFeasible()`, `relaxNodeInc()`, and `separate()`
- Two engines, LP and NLP
- We need a different node processor





Example 4: QP-Diving

- We solve a QP at each node (good warm-starting)
- Ocassionally solve NLP to find a better estimate of active constraints
- The constraints are linearizations of active constraints
- The QP objective is the gradient of the objective added to the hessian of Lagrangian
- IntVarHandler for `isFeasible()`, `getBranchingCandidates()` and `branch()`
- QPDProcessor for processing the nodes (different fathoming rules)
- QPDRelaxer for creating/updating QP approximations



Algorithms for Convex MINLPs
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BONMIN-QG
Minotaur BnB
QP-Diving
Minotaur QG

Closing Remarks

- Minotaur is a flexible objected-oriented open-source framework
- Its components can be combined to create powerful solvers . . .
- . . . at least for convex MINLPs
- We are working on some algorithms for nonconvex MINLPs also
- We would like YOU to try implement your ideas in Minotaur
- Latest 'Nightly' version of the source is available on the website
- An new stable version of binaries and libraries will be available soon

MINOTAUR

It's only half bull

http://wiki.mcs.anl.gov/minotaur