Optimal Steiner trees subject to mechanical constraints

or: Routing steam pipes in power plants

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2014-06-03
Discrete Optimization
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WIRTSCHAFTS MATHEMATIK

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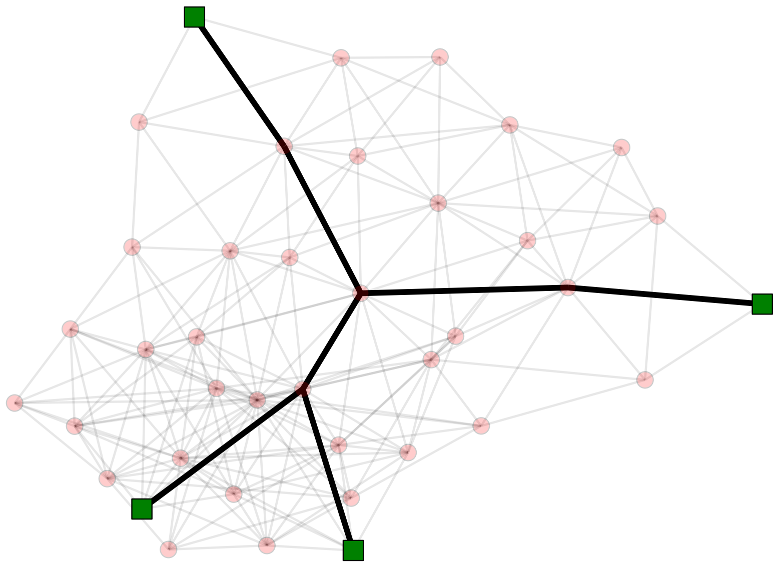
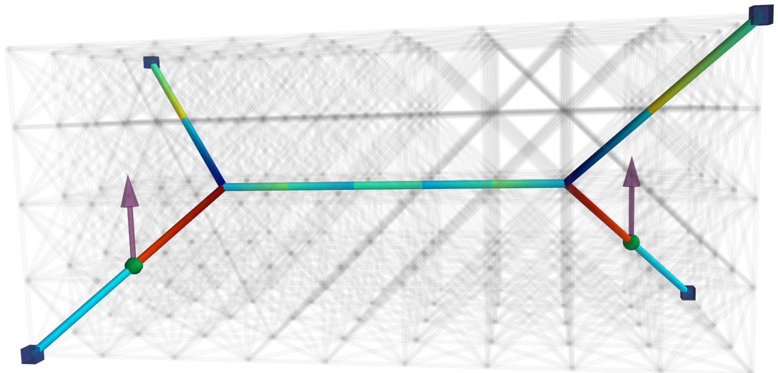
What’s our problem?

Why is it interesting?

Combinatorics: Path/Steiner tree



Nonlinearities: Mechanics



Our sponsors

- Part of **BMBF**-Project *LeOpIn* – Lifecycle oriented optimization for a resource- and energy-efficient infrastructure
- In cooperation with Bilfinger, sponsored by the German ministry of education and research
- With the groups of Michael Stingl, Günter Leugering and Eberhard Bänsch at FAU Erlangen-Nürnberg

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More about the application

Physical dimensions

- Power plant: $50\,\mathrm{m} \times 75\,\mathrm{m} \times 100\,\mathrm{m}$
- Pipe length: $\approx 450\,\mathrm{m}$ per section
- Pipe diameter: $770\,\mathrm{mm}$, wall thickness: $63\,\mathrm{mm}$
- Working conditions: $631\,^\circ\mathrm{C}$, $300\,\mathrm{bar}$

Requirements

- Mainly two types of cost: Operating costs and building costs
- Main constraint: Mechanical stresses must lie in tolerances
- Two scenarios: Cold and hot

Basic problem statement

$$\begin{aligned}
\min\ & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, \mathbf{u}(x, y)) \\
\text{s. t.}\ & \text{Steiner tree}(x) \\
& \text{pipe physics}(x, y, \mathbf{u}(x, y)) \\
& \text{hangers}(x, y, \mathbf{u}(x, y)) \\
& \text{industrial standards}(x, y, \mathbf{u}(x, y))
\end{aligned}$$

Variables

- x Pipe variables
- y Hanger variables
- $\mathbf{u}$ Displacement variables (depend on x and y)

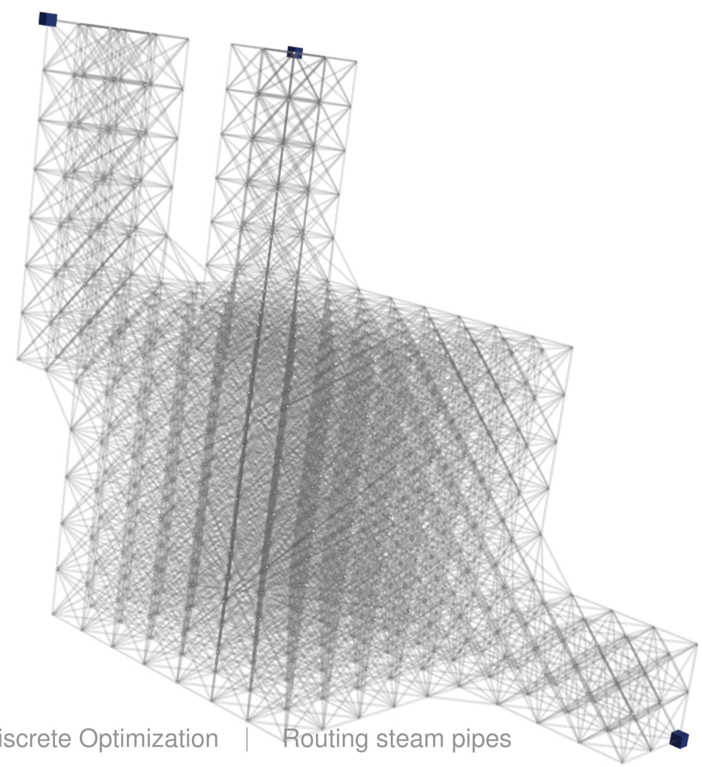
Steinertree model

- Huge catalogue of Steiner tree models available
- Only few terminals in our application $\rightarrow$ Use a flow formulation
- Computational study shows advantage over other models

Modeling the physics

Groundstructure approach

- Fixed and free nodes
- Potential elements between nodes
- Elements follow laws of linear elasticity



Linear Timoshenko beam

- Axial dimension is dominating length scale
- Cross-sections may rotate independently of the beam axis

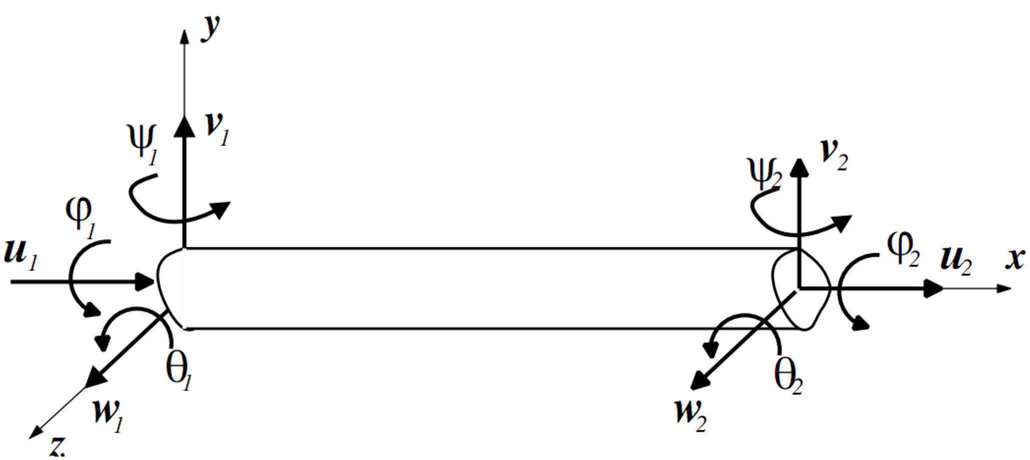


Figure : Source: Y. Lou, An Efficient 3D Timoshenko Beam Element with Consistent Shape Functions

- Variables for the displacement of the center line (u, v, w)
- Variables for the rotation of the cross-section (φ, θ, ψ)

Linear 3D Timoshenko beam equations

$$
\begin{aligned}
EAu'' + q_x &= 0\\
kGA(v'' - \theta') + q_y &= 0\\
kGA(w'' + \psi') + q_z &= 0\\
GI_t\varphi'' + m_x &= 0\\
EI_z\psi'' - kGA(w' + \psi) + m_y &= 0\\
EI_y\theta'' + kGA(v' - \theta) + m_z &= 0
\end{aligned}
$$

EA: extensional stiffness
kGA: shear stiffness with factor
GI_t: torsional stiffness
EI: bending stiffness

- Analytical solutions to homogeneous Timoshenko equations as ansatz functions give rise to proper stiffness matrices (Luo, 2008)
- Global stiffness matrix: $K(x) = \sum_{i=1}^{n} T_i^T K_i T_i x_i$

$$K(x)\mathbf{u} = \sum_{i=1}^{n} g_i x_i + \sum_{j=1}^{m} l_j h_j$$

Industrial standards (DIN EN 13480)

- Restrictions on the maximal supporting forces at fixed nodes
- Pipe must fulfill constraints based on moments
- Several constraints involve the difference of the moments from different loading scenarios

$$
\begin{aligned}
\mathbf{R}\mathbf{u} &\leqslant \mathbf{r} \\
M_e^h &= \hat{N}^h \mathbf{u}_e^h \\
M_e^c &= \hat{N}^c \mathbf{u}_e^c \\
M_e^D &= M_e^h - M_e^c \\
(M_{e1}^h)^2 + (M_{e2}^h)^2 + (M_{e3}^h)^2 &\leqslant (M_{\max}^h)^2 \\
(M_{e1}^D)^2 + (M_{e2}^D)^2 + (M_{e3}^D)^2 &\leqslant (M_{\max}^D)^2
\end{aligned}
$$

Solution approaches

$$\begin{aligned} \min\ & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, u(x,y)) \\ \text{s.t.}\ & \text{Steiner tree}(x) \\ & \text{pipe physics}(x, y, u(x,y)) \\ & \text{hangers}(x, y, u(x,y)) \\ & \text{industrial standards}(x, y, u(x,y)) \end{aligned}$$

MILP Model

- Linearize non-convex terms $z_{il} = x_i u_l$
- Can be done completely or adaptive

MISOCP Model

- Replace industrial standards with substitute constraint
- Problem becomes convex

Decomposition

- Decompose in master- and subproblem
- Both problems are convex

Decomposition approach

- General Idea: GBD
- Applied for truss problems: Muñoz and Stolpe [2011]

Masterproblem

$$\begin{aligned}\min\ & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, \mathbf{u}(x, y)) \\ \text{s.t.}\ & \text{Steinertree}(x)\end{aligned}$$

$$\longleftrightarrow$$

Subproblem for fixed $\hat{x}$

$$\begin{aligned}\min\ & c_{\text{hangers}}(y, \mathbf{u}(\hat{x}, y)) \\ \text{s.t.}\ & \text{pipe physics}(\hat{x}, y, \mathbf{u}(\hat{x}, y)) \\ & \text{hangers}(\hat{x}, y, \mathbf{u}(\hat{x}, y)) \\ & \text{industrial standards}(\hat{x}, y, \mathbf{u}(\hat{x}, y))\end{aligned}$$

Fast check for infeasibility

$$
\begin{aligned}
Ku &= b + \sum_{n \in \mathcal{V}} l_n h_n \\
\Leftrightarrow \qquad u &= K^{-1} b + \sum_{n \in \mathcal{V}} K^{-1} l_n h_n
\end{aligned}
$$

Simple test whether $\underline{u} \leqslant u \leqslant \bar{u}$ is attainable

If

$$\sum_{n \in \mathcal{V}} \max\left\{K^{-1} l_n h^{\text{max}}, 0\right\} < \underline{u} - K^{-1} b$$

or

$$\sum_{n \in \mathcal{V}} \min\left\{K^{-1} l_n h^{\text{max}}, 0\right\} > \bar{u} - K^{-1} b$$

holds, then the subproblem is infeasible.

Cutting planes for the subproblem

- To speed up subproblem we use cutting planes
- Derived by using MIR on single node sets on the equilibrium equation

$$\mathbf{u} = K^{-1}b + \sum_{n \in \mathcal{V}} K^{-1} l_n h_n$$

Still to do

We have no idea how to generalize this to larger node sets

Numerical examples

Name	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{E}\rvert$	$\lvert\mathcal{T}\rvert$	MIP (adaptive)		Decomposition		
				Time (s)	Gap (%)	Time (s)	# cb	Gap (%)
cube04	9	32	2	6.3	0	0.27	3	0
temptest	63	434	2	7200	41.7	515.6	2	0
BB-04	123	974	3	7200	27.8	7200	5074	0.07
BB-04_ff	141	964	3	7200	–	7200	5367	0.04
big-04	250	2241	4	7200	–	4249.8	10	0

Numerical examples