Recent progress in Nonlinear Network Design with applications to Water Networks

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joint work with Jesco Humpola and Andrea Lodi

MINLP2014 poster session, 06/02/2014

Modeling pressurized water networks

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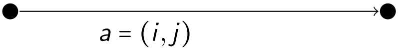
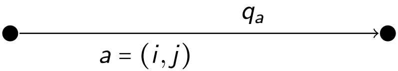


Modeling pressurized water networks



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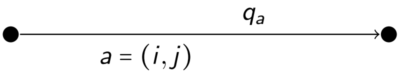

- Flow-conservation constraints at nodes j:

$$d_j = \sum_{a \in \delta_j^+} q_a - \sum_{a \in \delta_j^-} q_a$$

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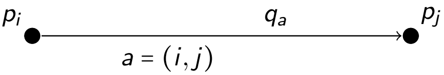

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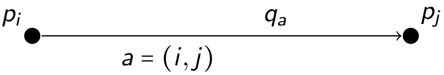

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$$d_j = \sum_{a \in \delta_j^+} q_a - \sum_{a \in \delta_j^-} q_a$$

- Potential-flow-coupling constraint (pressure-loss equation) on arcs a:

$$p_i - p_j = \Phi_a(q_a, D_a)$$

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Typical form of pressure-loss $\Phi_a(\cdot)$:

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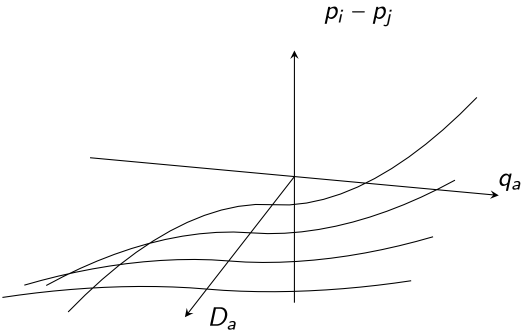

Non-convex overall model

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Non-convex overall model

$$d_j = \sum_{a \in \delta_j^+} q_a - \sum_{a \in \delta_j^-} q_a \qquad j \in \mathcal{N} \setminus \mathcal{S}$$

$$p_i - p_j = C_a \cdot L_a \cdot \frac{sign(q_a)|q_a|^{1.852}}{D_a^{4.87}} \qquad a = (i,j) \in \mathcal{A}$$

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Non-convex overall model

$$
\begin{array}{rll}
\text{minimize} & \displaystyle\sum_{a\in\mathcal{A}} L_a \sum_{i=1}^{r_a} C_{a,i} X_{a,i} & \\
\text{subject to} & \displaystyle d_j = \sum_{a\in\delta_j^+} q_a - \sum_{a\in\delta_j^-} q_a & j \in \mathcal{N} \setminus \mathcal{S} \\
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& D_a = \sum_{i=1}^{r_a} D_{a,i} X_{a,i} & a \in \mathcal{A} \\
& \sum_{i=1}^{r_a} X_{a,i} = 1 & a \in \mathcal{A} \\
& |q_a| \leq \frac{\pi}{4} v_a^{max} D_a^2 & a \in \mathcal{A} \\
& p_j = p_j^0 & j \in \mathcal{S} \\
& \underline{p}_j \leq p_i \leq \overline{p}_j & j \in \mathcal{N} \setminus \mathcal{S} \\
& X_{a,i} \in \{0,1\} & i = 1, \ldots, r_a, \;\; a \in \mathcal{A}
\end{aligned}
$$

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- Test the procedure on set of benchmark instances